XVI. The Bakerian Lecture.—*On the Radiation of Heat from the Moon, the Law of its Absorption by our Atmosphere, and of its Variation in Amount with her Phases. By the* Earl of Rosse, *F.R.S.*

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In the years 1869 and 1870 I communicated to the Royal Society the results of a series of experiments made with the view of determining, if possible, the amount of radiant heat coming to the earth from the moon in various conditions of phase, and the nature of that heat as regards the average refrangibility of the rays. Though more successful than I had at first been led to expect, the imperfect accordance between many of the observations still left much to be desired, and the novelty and importance of the subject appeared sufficient to render it advisable to pursue the investigation with greater care and closer attention to details than had hitherto been deemed necessary.

Since the conclusion of the series of observations which form the subject of the second paper above referred to, nothing (with the exception of a short series of observations in August and October 1870, of which mention is made towards the end of this paper) was done towards pursuing the subject till the spring of the following year (1871), when the series of observations which form the subject of the present paper were commenced, the same apparatus (only slightly modified) being used and the same method of observation adopted; but, with the view of obtaining an approximate value of the absorption of the moon's heat in its passage through our atmosphere, and of rendering possible the satisfactory comparison of observations made at different zenith-distances of the moon, the observations were in many cases carried on at intervals at all possible zenith-distances on the same night, and the most favourable opportunities for observing the moon at very different zenith-distances in various conditions of the atmosphere were not lost.

Before proceeding further it may be well to mention the small changes in the apparatus already referred to.

The piles instead of being, as before, placed in two separate circuits or loops, united close to the terminals of the galvanometer, have, since the beginning of August 1870, been placed in one and the same circuit, the position of the poles of the second pile relative to the galvanometer-circuit being, as before, the reverse of the first; and further, to protect the piles more effectually from draughts of air, they and the small concave mirrors were enclosed in a box with glass* sides, that side next the large speculum being still (unless otherwise specified) left open. The piles were single pairs of small cross section of the kind described in the 'Proceedings of the Royal Society,' No. 122, 1870,

* Of glass to enable the assistant to see when the moon's image was central on each concave mirror.

and their back solderings were protected from sudden changes of temperature by being imbedded in wax. More care has been taken to protect the apparatus from the wind; and with that view, during the last two years, the cloth which had been hitherto used to cover the lattice-tube was prolonged to about 5 feet beyond the end of the tube, or 6 feet outside the piles. The three-foot telescope was employed in these as in the former experiments.

The observations were made by directing the telescope towards the moon, so that by a slight motion the faces of the piles were alternately exposed for a period of one minute to the radiation from the part of the sky containing the moon's disk, while the other pile was exposed to that from an adjacent circle of sky. The galvanometer was read off just before each motion of the telescope. Thus the difference between two successive readings of the galvanometer exhibited the heat-effect on double the scale that would have been possible with one pile. After three or four preliminary settings of the telescope had been made, to give the assurance that every thing was in working order and the needle of the galvanometer vibrating in equal arcs on each side of the zero, eleven readings of the galvanometer, giving ten differences, were taken in succession. The sidereal time was noted and the altitude read off on a quadrant attached to the telescope, graduated to half degrees, and allowing the estimation of tenths. As far as was at all compatible with the progress of the observations all cloud was avoided, and no readings were taken unless the sky near the moon was clear or very nearly so. The zenith-distance for the middle of each set of ten differences was obtained by a graphical process from the altitudes actually observed, and the arithmetical mean (G) of the differences was taken as the corresponding heat-effect.

These quantities, and others of which an explanation is given further on, are entered in the following Table.

Journal of Observations and preliminary Reductions.

We have:—

In column III. the observed deviation of the needle.

In column IV. the moon's distance from the point directly opposite to the sun.

In column VII. the logarithm of the factor for reducing the galvanometer-readings from *tangent* to *arc*, it being assumed that the latter was proportional to the heating-effect.

In columns VIII. & IX. the respective logarithms for reducing each set of readings for change of phase and of the moon's apparent semidiameter to what it would have been had the observation been made at the time and under the circumstances given in columns XI., XII., & XIII.; so that we obtained the heating-effects (given in column X.) of the moon at various zenith-distances on each night, independent of all other causes of variation.

The remaining columns require no explanation, beyond what will be found in the pages which follow the Table.

TABLE.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
			The Moon's							The adopted mean		
1871.	Sidereal time.	G.	ε.	Apparent zenith-distance.	Apparent semi-diameter.	log (g).	log (ε).	log (σ).	log (G corr.).	Sidereal time.	ε.	Apparent semi-diameter.
	h m		° ′	°	′ ″					h m	° ′	′ ″
Mar. 2.	12 22	92·9	− 50 4	61·4	15 19·8	9·9998	0·0027	9·9982	1·9687	13 20·0	− 49 36	15 18·0
*	13 12	86·2	− 49 41	68·7	15 18·1	9·9999	0·0005	0·0000	1·9359			
	13 43	72·9	− 49 26	73·0	15 17·1	9·9999	9·9990	0·0008	1·8624			
	14 8	65·4	− 49 14	76·6	15 16·1	9·9999	9·9981	0·0018	1·8154			
Mar. 27.	7 45	34·2	−106 8	42·5	14 59·5	0·0000	0·0127	9·9966	1·5433	10 0·0	−105 19	14 56·0
	8 8	36·9	−106 0	45·3	14 59·1	0·0000	0·0106	9·9970	1·5746			
	8 30	33·3	−105 53	47·8	14 58·6	0·0000	0·0089	9·9974	1·5287			
	8 58	31·5	−105 43	51·7	14 57·9	0·0000	0·0063	9·9982	1·5028			
	9 24	31·3	−105 33	55·1	14 57·2	0·0000	0·0031	9·9988	1·4974			
	10 12	25·8	−105 14	62·2	14 55·7	0·0000	9·9988	0·0004	1·4108			
	10 34	23·3	−105 4	65·3	14 55·0	0·0000	9·9962	0·0010	1·3646			
	11 16	21·9	−104 46	71·4	14 53·6	0·0000	9·9917	0·0022	1·3343			
	11 37	20·9	−104 37	74·4	14 52·9	0·0000	9·9895	0·0030	1·3126			
	11 59	17·6	−104 25	77·5	14 52·2	0·0000	9·9865	0·0036	1·2356			
Mar. 28.	8 57	40·8	− 94 46	43·3	15 4·4	0·0000	0·0097	9·9966	1·6170	11 0·0	− 94 0	15 1·0
	9 17	40·8	− 94 38	46·1	15 3·9	0·0000	0·0080	9·9972	1·6159			
	10 55	42·1	− 94 2	59·9	15 1·1	0·0000	0·0004	0·0000	1·6247			
	11 13	39·7	− 93 55	62·5	15 0·6	0·0000	9·9990	0·0004	1·5982			
	11 30	34·8	− 93 47	64·9	15 0·0	0·0000	9·9973	0·0010	1·5399			
	11 48	35·2	− 93 40	67·6	14 59·5	0·0000	9·9958	0·0014	1·5437			
	12 26	32·9	− 93 22	73·0	14 58·1	0·0000	9·9920	0·0028	1·5120			
	12 42	34·6	− 93 14	75·2	14 57·7	0·0000	9·9903	0·0032	1·5326			
Mar. 29.	9 16	66·0	− 83 30	39·0	15 13·2	9·9999	0·0101	9·9962	1·8257	11 46·0	− 82 33	15 9·2
	9 35	65·9	− 83 23	41·4	15 12·8	9·9999	0·0088	9·9966	1·8242			
	9 52	64·4	− 83 18	43·5	15 12·4	9·9999	0·0080	9·9970	1·8138			
	11 44	56·3	− 82 34	58·9	15 9·4	9·9999	0·0002	9·9998	1·7504			
	12 3	55·0	− 82 26	61·8	15 8·7	9·9999	9·9988	0·0006	1·7397			
	12 27	56·1	− 82 15	65·2	15 8·0	9·9999	9·9968	0·0012	1·7469			
	12 43	56·1	− 82 8	67·5	15 7·4	9·9999	9·9956	0·0018	1·7462			
	13 5	46·6	− 81 58	70·7	15 6·6	9·9999	9·9938	0·0026	1·6647			
	13 31	39·8	− 81 46	74·3	15 5·8	9·9999	9·9917	0·0034	1·5949			
	13 49	34·2	− 81 37	76·8	15 5·2	9·9999	9·9901	0·0040	1·5280			
Apr. 3.	10 53	315·5	− 22 49	46·6	16 18·1	9·9983	0·0034	9·9970	2·4977	14 0·0	− 21 31	16 14·4
†	11 27	318·7	− 22 35	46·0	16 18·2	9·9982	0·0028	9·9968	2·5012			
	12 8	327·5	− 22 18	46·4	16 18·1	9·9981	0·0021	9·9970	2·5124			
	12 47	318·2	− 22 3	48·2	16 17·5	9·9982	0·0014	9·9974	2·4997			
	13 55	303·2	− 21 35	53·9	16 16·4	9·9984	0·0001	9·9984	2·4786			
	15 3	315·5	− 21 2	62·1	16 14·4	9·9983	9·9987	0·0000	2·4960			
	15 44	251·7	− 20 41	67·5	16 13·0	9·9989	9·9978	0·0014	2·3990			
	(16 30	166·4)	haze									
	16 45	181·9	− 20 11	75·8	16 10·7	9·9994	9·9965	0·0034	2·2591			
Apr. 30.	12 14	204·3	− 54 42	45·1	16 4·9	9·9993	0·0060	9·9960	2·3115	14 30·0	− 53 46	16 0·5
	12 52	212·1	− 54 28	47·8	16 4·1	9·9992	0·0042	9·9968	2·3267			
	13 55	197·9	− 54 1	54·2	16 2·8	9·9993	0·0015	9·9978	2·2951			
	15 32	165·3	− 53 17	67·1	15 59·6	9·9995	9·9969	0·0008	2·2155			
	16 16	148·2	− 52 55	73·4	15 57·9	9·9996	9·9946	0·0024	2·1675			
	16 56	138·6	− 52 35	78·8	15 56·4	9·9997	9·9925	0·0036	2·1376			
May 3.	11 47	298·2	− 14 49	66·3	16 40·1	9·9984	0·0010	0·0004	2·4743	13 0·0	− 14 16	16 40·6
	12 48	324·6	− 14 21	62·3	16 41·2	9·9982	0·0002	9·9995	2·5092			

* On March 24th, 1871, the moon was observed at an altitude of 18°; no trace of heat was perceptible. Moon's age 3·6 days, 138° from full.

† Observations were also made on April 6 and April 7, 1871; but as they were intentionally made under most unfavourable circumstances to test the effect of wind and haze, they are not included in the above journal.

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TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
			The Moon's							The adopted mean		
1871.	Sidereal time.	G.	ε.	Apparent zenith-distance.	Apparent semi-diameter.	log (g).	log (ε).	log (σ).	log (G corr.).	Sidereal time.	ε.	Apparent semi-diameter.
	h m									h m		
May 28.	14 50	88·0	−72° 48′	59°·8	15′ 57″·9	9·9999	0·0050	9·9974	1·9468	16 0·0	−72° 14′	15′ 55″·3
	15 32	80·7	−72 29	65·2	15 56·6	9·9999	0·0025	9·9986	1·9079			
	16 7	75·5	−72 11	70·1	15 55·3	9·9999	9·9995	0·0000	1·8773			
	16 40	59·1	−71 55	74·9	15 54·0	9·9999	9·9972	0·0012	1·7699			
	16 58	51·4	−71 46	77·7	15 53·2	0·0000	9·9959	0·0018	1·7087			
	17 12	47·1	−71 39	79·6	15 52·7	0·0000	9·9949	0·0024	1·6703			
May 29.	14 26	121·4	−60 37	57·0	16 13·6	9·9997	0·0050	9·9972	2·0861	16 0·0	−59 55	16 10·7
	15 0	120·9	−60 22	60·2	16 12·8	9·9997	0·0032	9·9980	2·0834			
	15 35	106·4	−60 11	64·1	16 11·8	9·9998	0·0019	9·9990	2·0277			
	17 25	61·2	−59 14	78·5	16 7·8	9·9999	9·9951	0·0026	1·7844			
May 30.	16 22	138·3	−45 57	68·3	16 26·9	9·9997	0·0018	9·9988	2·1412	17 0·0	−45 37	16 25·7
	17 30	108·5	−45 24	76·6	16 24·5	9·9998	9·9988	0·0010	2·0351			
June 1.	15 56	279·1	−18 10	69·7	16 48·0	9·9986	0·0014	9·9988	2·4446	17 30·0	−17 29	16 46·5
	16 34	273·7	−17 53	71·0	16 47·7	9·9987	0·0008	9·9994	2·4362			
	17 13	257·2	−17 36	73·3	16 47·0	9·9988	0·0003	9·9996	2·4088			
	18 25	193·2	−17 4	79·4	16 45·1	9·9993	9·9991	0·0012	2·2856			
June 2.	15 47	327·9	− 3 59	74·3	16 50·5	9·9981	0·0001	0·0000	2·5139	16 25·0	− 3 42	16 50·5
*	16 25	285·1	− 3 42	73·8	16 50·6	9·9986	0·0000	9·9999	2·4535			
	17 2	332·5	− 3 26	74·2	16 50·5	9·9981	9·9999	0·0000	2·5197			
Nov. 1.	1 52	139·3	+51 48	55·3	14 54·0	9·9997	9·9986	0·0012	2·1435	2 30·0	+52 2	14 55·3
	2 8	130·9	+51 54	52·8	14 54·5	9·9997	9·9992	0·0008	2·1167			
	2 29	141·3	+52 2	49·9	14 55·1	9·9996	0·0000	0·0002	2·1499			
	3 31	152·2	+52 24	41·5	14 56·6	9·9996	0·0023	9·9986	2·1829			
Dec. 19.	2 21	87·4	−78 26	57·8	15 30·8	9·9999	0·0010	9·9996	1·9420	2 36·0	−78 20	15 30·4
	2 37	81·8	−78 20	59·6	15 30·3	9·9999	0·0000	0·0002	1·9129			
	2 51	79·9	−78 14	60·7	15 30·1	9·9999	9·9990	0·0004	1·9018			
Dec. 20.	1 5	121·9	−67 9	49·9	15 20·6	9·9997	0·0082	9·9970	2·0910	4 0·0	−66 8	15 17·4
	1 36	123·8	−66 58	49·5	15 20·7	9·9997	0·0067	9·9970	2·0961			
	1 51	132·3	−66 53	49·7	15 20·6	9·9997	0·0061	9·9970	2·1244			
	2 6	124·0	−66 48	50·0	15 20·6	9·9997	0·0054	9·9970	2·0955			
	5 31	103·5	−65 32	68·5	15 16·5	9·9998	9·9953	0·0010	2·0111			
	6 25	88·4	−65 9	75·8	15 14·7	9·9999	9·9922	0·0028	1·9414			
	6 39	80·5	−65 3	77·8	15 14·2	9·9999	9·9914	0·0032	1·9003			
Dec. 25.	2 8	356·1	−10 56	48·3	14 53·5	9·9978	0·0009	0·0014	2·5516	4 0·0	−10 16	14 55·0
	2 40	359·7	−10 44	44·0	14 54·2	9·9977	0·0007	0·0008	2·5552			
	3 14	357·3	−10 32	39·8	14 54·9	9·9978	0·0003	0·0000	2·5512			
	3 47	370·1	−10 20	36·1	14 55·5	9·9976	0·0001	9·9996	2·5656			
	4 26	387·6	−10 8	32·7	14 56·0	9·9974	9·9999	9·9990	2·5847			
	5 2	379·7	− 9 57	30·7	14 56·2	9·9975	9·9996	9·9988	2·5753			
	5 17	384·6	− 9 53	30·0	14 56·3	9·9974	9·9995	9·9988	2·5807			
Dec. 26.	1 57	315·4	± 0 36	56·2	14 51·4	9·9983	0·0000	0·0016	2·4988	3 0·0	+ 0 45	14 53·0
†	2 36	308·0	∓ 0 41	47·8	14 52·9	9·9983	0·0000	0·0000	2·4869			
	6 50	362·0	+ 1 22	29·3	14 55·8	9·9977	0·0001	9·9974	2·5539			

* 1871, June 2.—Full moon; the moon passed clear of the earth's shadow and penumbra.

† 1871, December 26.—Full moon; using the formula given at p. 135 of the Appendix to the Nautical Almanac for 1836, which makes the semidiameter of the earth's penumbra equal to $\frac{61}{60}(P'+\pi-\sigma)+2\sigma$ (where P′ is the moon's horizontal parallax, π the sun's parallax, and σ the sun's semidiameter), it will be found that at $3^h\ 7^m$ sidereal time the semidiameter of the penumbra was 1° 11′ 2″, while the distance of the centre of the moon from the centre of the shadow was 1° 10′ 42″; so that an immersion to the extent of 20″ only took place, but at the time of observation the moon was clear of the penumbra.

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
1871.	Sidereal time.	G.	The Moon's ε.	The Moon's Apparent zenith-distance.	The Moon's Apparent semi-diameter.	log (*g*).	log (ε).	log (σ).	log (G corr.).	The adopted mean Sidereal time.	The adopted mean ε.	The adopted mean Apparent semi-diameter.
	h m		° ′	°	′ ″					h m	° ′	′ ″
Dec. 28.	4 36	306·6	+ 22 49	49·9	14 56·2	9·9984	9·9977	0·0018	2·4845	7 0·0	+ 23 38	14 58·0
	5 9	297·3	+ 23 1	45·4	14 56·9	9·9984	9·9982	0·0010	2·4708			
	5 50	299·7	+ 23 16	40·3	14 57·7	9·9984	9·9989	0·0004	2·4744			
	6 19	298·7	+ 23 25	37·2	14 58·1	9·9984	9·9993	0·0000	2·4729			
	8 52	288·7	+ 24 12	31·1	14 59·0	9·9985	0·0016	9·9990	2·4596			
	9 26	274·7	+ 24 24	33·3	14 58·7	9·9987	0·0022	9·9992	2·4390			
	10 3	274·3	+ 24 37	36·5	14 58·3	9·9987	0·0028	9·9998	2·4396			
1872.												
Jan. 2.	12 39	93·8	+ 82 26	51·0	15 37·5	9·9998	9·9993	0·0000	1·9713	12 50·0	+ 82 30	15 37·5
	12 50	85·2	+ 82 30	51·2	15 37·5	9·9999	0·0000	0·0000	1·9303			
Jan. 5.	14 16	17·7	+119 46	68·2	16 17·8	0·0000	9·9984	0·0000	1·2464	14 32·0	+119 51	16 17·8
	14 32	18·5	+119 51	67·9	16 17·8	0·0000	0·0000	0·0000	1·2672			
Jan. 18.	4 29	102·7	− 74 43	45·4	15 11·8	9·9998	0·0049	9·9982	2·0145	6 0·0	− 74 11	15 10·0
*	4 45	103·1	− 74 37	46·8	15 11·5	9·9998	0·0040	9·9986	2·0156			
*	4 58	98·8	− 74 33	48·0	15 11·3	9·9998	0·0034	9·9988	1·9968			
	5 57	99·0	− 74 12	54·7	15 10·0	9·9998	0·0002	0·0000	1·9956			
	6 12	95·9	− 74 6	56·6	15 9·6	9·9998	9·9993	0·0002	1·9811			
*	6 24	90·0	− 74 2	58·2	15 9·3	9·9999	9·9980	0·0006	1·9527			
	7 36	86·7	− 73 34	68·2	15 7·0	9·9999	9·9943	0·0028	1·9350			
Jan. 19.	5 34	136·8	− 63 10	42·9	15 4·5	9·9997	0·0003	0·0000	2·1361	5 42·0	− 63 7·5	15 4·4
	5 50	125·6	− 63 5	44·6	15 4·2	9·9997	9·9996	0·0002	2·0985			
Jan. 20.	8 57	185·5	− 50 59	59·8	14 55·1	9·9994	0·0003	9·9998	2·2677	9 5·0	− 50 56	14 54·9
†	9 13	175·6	− 50 53	62·0	14 54·7	9·9995	9·9997	0·0002	2·2438			
Jan. 22.	9 55	163·6	− 28 54	50·1	14 53·4	9·9995	0·0006	9·9992	2·2131	10 20·0	− 28 44	14 52·7
	10 23	176·6	− 28 43	54·2	14 52·7	9·9995	9·9999	0·0000	2·2464			
	10 40	188·4	− 28 37	57·2	14 52·0	9·9994	9·9996	0·0006	2·2747			
Jan. 27.	5 16	247·6	+ 25 19	70·4	15 16·1	9·9989	....	....	2·3927	5 16·0	+ 25 19	15 16·1
Jan. 30.	9 17	116·4	+ 61 40	69·8	15 32·2	9·9998	9·9985	0·0010	2·0653	9 48·0	+ 61 53	15 33·2
	9 32	128·1	+ 61 46	68·0	15 32·7	9·9997	9·9992	0·0006	2·1070			
	9 47	130·0	+ 61 53	66·3	15 33·1	9·9997	0·0000	0·0002	2·1138			
‡	10 1	118·3	+ 61 58	64·8	15 33·5	9·9997	0·0007	9·9998	2·0732			
	10 16	130·4	+ 62 4	63·3	15 33·9	9·9997	0·0014	9·9994	2·1157			
‡	10 27	130·1	+ 62 9	62·2	15 34·1	9·9997	0·0020	9·9994	2·1153			
Feb. 1.	14 12	70·7	+ 87 54	66·1	15 57·7	9·9999	....	....	1·8493	14 12·0	+ 87 54·0	15 57·7
Feb. 16.	5 53	88·1	− 83 46	39·4	15 6·5	9·9999	0·0063	9·9974	1·9486	7 30·0	− 83 12	15 3·5
	6 14	80·2	− 83 39	41·7	15 6·1	9·9999	0·0051	9·9976	1·9068			
	6 27	75·2	− 83 35	43·1	15 5·9	9·9999	0·0043	9·9976	1·8780			
	6 43	77·1	− 83 29	45·0	15 5·5	9·9999	0·0033	9·9982	1·8885			
	9 15	65·8	− 82 32	65·7	15 1·2	9·9999	9·9931	0·0022	1·8134			
	9 28	62·2	− 82 26	67·5	15 0·8	9·9999	9·9920	0·0026	1·7884			
Feb. 20.	12 33	197·6	− 38 21	60·7	14 55·6	9·9993	0·0009	9·9994	2·2954	13 0·0	− 38 9	14 55·0
	12 47	183·1	− 38 15	62·7	14 55·3	9·9994	0·0004	9·9998	2·2623			
	13 0	174·7	− 38 9	64·5	14 54·9	9·9995	0·0000	0·0002	2·2419			
	13 14	171·7	− 38 2	66·4	14 54·4	9·9995	9·9995	0·0006	2·2343			

* 1872, January 18.—On this night an iron candle-holder stood near the galvanometer; experiments made March 13 show, however, that it did not affect the value of the readings although it considerably altered the zero. The sets at $4^h\ 58^m$ and $6^h\ 24^m$ consisted only of eight and five differences respectively; but as these were made under favourable circumstances, full weight has been given to them.

† January 20.—The scale was 36·5 in. from the mirror of the galvanometer instead of at the usual distance of 36·0 in.; therefore the readings have been multiplied by $\frac{36 \cdot 0}{36 \cdot 5}$.

‡ January 30.—$10^h\ 1^m$, barely perceptible haze $10^h\ 27^m$, very light cloud again.

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
1872.	Sidereal time.	G.	The Moon's ε.	The Moon's Apparent zenith-distance.	The Moon's Apparent semi-diameter.	log (g).	log (ε).	log (σ).	log (G corr.).	The adopted mean Sidereal time.	The adopted mean ε.	The adopted mean Apparent semi-diameter.
	h m		° ′	°	′ ″					h m	° ′	′ ″
Feb. 24.	9 19	347·5	+ 7 57	46·7	15 22·0	9·9979	9·9993	0·0000	2·5381	11 30·0	+ 8 33	15 22·0
	9 34	341·5	+ 8 0	45·5	15 22·2	9·9980	9·9994	9·9998	2·5306			
	10 10	334·0	+ 8 11	43·4	15 22·5	9·9980	9·9996	9·9996	2·5209			
	11 34	314·5	+ 8 33	42·8	15 22·7	9·9983	0·0000	9·9994	2·4953			
	11 48	313·7	+ 8 38	43·3	15 22·6	9·9983	0·0001	9·9996	2·4945			
	12 2	315·8	+ 8 42	44·0	15 22·5	9·9983	0·0002	9·9996	2·4975			
	12 16	306·2	+ 8 46	44·8	15 22·4	9·9984	0·0002	9·9996	2·4842			
	12 53	274·4	+ 8 58	47·7	15 21·8	9·9987	0·0005	0·0000	2·4376			
Mar. 21.	10 11	298·9	− 36 56	36·3	15 15·4	9·9984	0·0010	9·9996	2·4746	10 45·0	−36 42	15 15·1
	10 44	296·7	− 36 42	37·4	15 15·2	9·9985	0·0000	9·9998	2·4706			
	11 22	282·1	− 36 29	39·8	15 14·8	9·9986	9·9992	0·0002	2·4484			
Mar. 22.	11 57	290·0	− 24 50	42·9	15 23·7	9·9985	0·0011	9·9992	2·4612	13 0·0	−24 27	15 22·8
	12 11	285·5	− 24 46	44·1	15 23·5	9·9986	0·0009	9·9994	2·4545			
	12 25	290·3	− 24 40	45·3	15 23·4	9·9985	0·0006	9·9994	2·4613			
	12 57	278·2	− 24 28	48·2	15 22·7	9·9986	0·0000	0·0002	2·4431			
	13 48	265·5	− 24 8	54·3	15 22·3	9·9988	9·9990	0·0012	2·4230			
	14 4	258·6	− 24 0	56·3	15 21·0	9·9988	9·9986	0·0016	2·4116			
Mar. 24.	9 48	354·3	− 4 26	59·3	15 39·3	9·9978	0·0001	0·0006	2·5479	12 0·0	− 4 9	15 40·0
*	11 47	361·8	− 4 10	51·5	15 41·0	9·9977	0·0000	9·9990	2·5552			
	13 37	364·1	− 4 1	53·5	15 40·6	9·9977	9·9999	9·9994	2·5582			
	13 51	363·5	− 4 0	54·5	15 40·4	9·9977	9·9999	9·9996	2·5577			
	14 21	358·5	− 4 0	56·9	15 39·9	9·9977	9·9999	0·0000	2·5521			
Mar. 26.	11 52	251·3	+ 24 44	68·1	15 54·6	9·9989	....	....	2·3991	11 52·0	+24 44	15 54·6
Apr. 15.	12 21	57·7	− 89 53	57·1	14 56·9	9·9999	0·0012	9·9996	1·7619	12 36·0	−89 47	14 56·4
	12 36	50·4	− 89 47	59·3	14 56·4	0·0000	0·0000	0·0000	1·7024			
	12 50	52·4	− 89 41	61·4	14 56·0	0·0000	9·9988	0·0004	1·7185			
Apr. 18.	10 57	175·1	− 56 55	38·8	15 19·1	9·9995	0·0050	9·9972	2·2449	13 0·0	−56 11	15 16·0
	11 53	166·1	− 56 35	42·2	15 18·6	9·9995	0·0027	9·9976	2·2202			
	12 26	161·4	− 56 24	45·3	15 18·1	9·9995	0·0015	9·9982	2·2071			
	12 41	164·2	− 56 18	46·8	15 17·7	9·9995	0·0013	9·9984	2·2145			
	13 57	159·7	− 55 48	56·3	15 15·8	9·9996	9·9975	0·0002	2·2007			
	14 11	154·1	− 55 42	58·3	15 15·4	9·9996	9·9968	0·0006	2·1848			
	14 25	156·9	− 55 37	60·3	15 14·9	9·9996	9·9963	0·0012	2·1927			
	14 53	147·1	− 55 24	64·4	15 14·0	9·9996	9·9948	0·0020	2·1640			
	15 9	129·8	− 55 17	66·7	15 13·4	9·9997	9·9941	0·0024	2·1096			

Determination of the Law of Extinction of the Moon's Radiant Heat in our Atmosphere.

As, apart from any *à priori* consideration, an examination of the former observations had shown that the quantity of heat reaching us from the moon was dependent upon her zenith-distance, it was necessary that the whole of the new observations should be carefully examined in order to determine the law of this dependence. But since the moon's phase and distance from the place of observation and from the sun's centre are constantly changing, the observations had to be freed from the first two of these sources of change before they could be used for deducing the corrections dependent upon the zenith-distance.

The necessary elements for the corrections were obtained as follows. The moon's

* March 24.—The sky rather hazy all night, but the observations were only taken during the clearer intervals.

tabular place was taken from the Nautical Almanac for every hour of sidereal time over which the observations extended; then the moon's parallax was calculated, also for every hour, by the following approximate formulæ*:—

$$\left.\begin{aligned} \alpha'-\alpha &= -[0\cdot3815]\,P\sin(\Theta-\alpha)\sec\delta \\ \tan\gamma &= \frac{[0\cdot1215]}{\cos(\Theta-\alpha)}, \\ \delta'-\delta &= -\frac{[9\cdot9009]\sin(\gamma-\delta)}{\sin\gamma}\,P, \end{aligned}\right\}$$

where α' = moon's apparent right ascension,

α = moon's true right ascension,

δ' = moon's apparent declination,

δ = moon's true declination,

P = moon's equatorial horizontal parallax,

Θ = sidereal time,

and $[0\cdot3815] = \log\frac{\rho'\cos\phi'}{15}$,

$[0\cdot1215] = \log\tan\phi'$,

$[9\cdot9009] = \log\rho'\sin\phi'$,

where ρ' = distance from centre of the earth,

ϕ' = geocentric latitude.

From the apparent places so obtained the moon's apparent elongation (ε) from the point opposite the sun was derived by the equation

$$\cos(\pi-\varepsilon)=\sin D\sin\delta'+\cos D\cos\delta'\cos(A-\alpha'),$$

where

D = sun's declination,

A = sun's right ascension.

It is evident that $\pi-\varepsilon$ represents very nearly the angular amount of the moon's apparent illuminated phase. A slight inaccuracy arises from neglecting the angle at the sun† in the plane triangle earth, sun, moon.

The moon's tabular semidiameter and a Table of augmentations gave her apparent semidiameter, σ. Finally, the moon's apparent altitude was taken from a Table of double entry calculated for the latitude of Birr Castle, to test, by its agreement with the readings of the quadrant, the accuracy of the calculations mentioned above.

In the absence of an exact knowledge of the law of variation of the moon's radiant heat with her change of phase, that enunciated at page 439, No. 112 of the 'Proceedings

* Brünnow's 'Spherical Astronomy,' page 150 (English edition, 1865).

† This angle never exceeds 9′ even at quadrature.

of the Royal Society,' 1869, was adopted*. In accordance with this law, if the radiant heat of the full moon be represented by unity, the heat (h) at any other phase will be (*cœteris paribus*)

$$h=\frac{(\pi-\varepsilon)\cos\varepsilon+\sin\varepsilon}{\pi}.$$

The logarithmic factors for reducing observations made at any elongation to a mean value (ε_0) for the night were readily derived from a Table of log h for every degree of ε.

As the moon's heating-power doubtless varies (*cœteris paribus*) as the square of her apparent semidiameter, the double of the logarithm of the apparent semidiameter at different times affords the simplest form of reduction for change of distance. In this way the logarithmic factors for correcting for changes of ε and σ in columns log (ε) and log (σ) were obtained.

For the larger readings of the galvanometer there is a small correction required to reduce the readings from tangent to arc†. This correction may be conveniently brought into the form of a factor if we assume, as was approximately the case, especially for the larger readings, that the amplitudes were equally large on each side of the zero-point‡. For if n is the difference of two readings and r the distance of the scale from the galvanometer, the correcting factor will be $\left(1-\frac{1}{12}\frac{n^2}{r^2}\right)$. In our case $r=1440$, and the log factor becomes

$$\text{log corr. } (g)=\log\left(1-\frac{n^2}{24883200}\right).$$

The values of log corr. (g) are given in column log (g) for each observation.

The sum of log G, log (g), log (ε), and log (σ) gives column log (G corr.), the logarithm

* There is a misprint in formula (a) at the place referred to; it should have been

$$Q=100\left[\left(1-\frac{\epsilon}{\pi}\right)\cos\epsilon+\frac{\sin\epsilon}{\pi}\right].$$

Since the publication of that paper it has been found that Lambert, in his 'Photometria,' had already made use of it in the form $c=\frac{2(\sin v-v\,.\cos v)\,.\,A\,.\sin^2 s\,.\sin^2\sigma\,.\,C}{3\pi\,.\sin^2 S}$,

where A = mean "albedo" of moon's surface,

s = sun's apparent semidiameter as seen at the moon,

σ = moon's apparent semidiameter,

C = brightness of an absolutely white plane illuminated by perpendicularly incident rays of the sun,

c = brightness of an absolutely white plane similarly illuminated by the rays of the moon,

v = elongation from the sun $=180°-\epsilon$,

S = sun's semidiameter as seen from the earth.

† It appears rather doubtful whether this small correction (which only amounts at most to $\frac{1}{200}$ of the whole heating-effect) should have been made at all, as from subsequent experiment the deviations in different parts of the scale were found to be very nearly identical with the same heating-effect; in fact the difference was less than the probable error of observation.

‡ Strictly speaking this correction should have been applied to each single reading, not to the mean of a set of readings, as was done to save labour.

of G corrected for change of phase, change of distance, and for the variation of the values of the scale.

These corrected values of log G are now available for the determination of the law of the extinction of the moon's radiant heat in the earth's atmosphere. For this purpose a graphic method, suggested by that employed by SEIDEL* in his researches concerning the extinction of stellar light, was made use of. The night's observations were arranged in order of zenith-distance, then from the log (G corr.) corresponding to the least zenith-distance on any given night each other log (G corr.) of that night was subtracted. The logarithmic differences so obtained were then entered on a sheet of cross-lined paper as ordinates, the zenith-distances being taken as abscissæ; a curve drawn through these points gave what might be called the reducing curve in zenith-distance for that night. A second night's observations were then treated in the same way, and the resulting curve moved up or down until it intersected the first night's curve at the middle of their overlapping parts. A third curve corresponding to a third night was then drawn and made to agree as nearly as possible with the mean of the curves already laid down; in this way thirty-three curves representing the logarithmic differences of the moon's radiant heat at different zenith-distances were laid down.

The mean of the differences in each degree of zenith-distance was now entered as a point at its corresponding mean zenith-distance on a separate sheet of paper, the number of observations being indicated by short radiating lines, in the way often adopted for star magnitudes, a simple dot representing a single observation. The ordinates of a curve drawn through or near these points gave a provisional Table for ϕz, or the logarithm that must be added to that of the reading at any zenith-distance to obtain the logarithm of the reading as it would have been shown had the moon been in the zenith. The Table thus deduced for the extinction of the moon's heat is here given in an abbreviated form.

Zen.-dist.	ϕz.
29°	0·0070
30	0·0071
35	0·0089
40	0·0124
45	0·0189
50	0·0296
55	0·0447
60	0·0665
65	0·1025
70	0·1525
75	0·2200
79	0·3079

* 'Untersuchungen über die gegenseitige Helligkeit der Fixsterne erster Grösse und über die Extinction des Lichtes in der Atmosphäre. Nebst einem Anhange über die Helligkeit der Sonne verglichen mit Sternen, und über die Licht reflectirende Kraft der Planeten.' Von LUDWIG SEIDEL. Munich, 1852.

As the moon was not observed at a less zenith-distance than 29°, SEIDEL'S value of φz for 29° was adopted: it is almost needless to remark that all the observations might equally well have been reduced to 29° zenith-distance as a superior limit; the only effect would have been to lessen all the subsequent numbers of this paper by $\frac{1}{60}$ part. With the help of the above Table all the observations were reduced to the zenith, and a mean zenith value for each night obtained. From these mean zenith quantities the mean values corresponding to the altitudes at which observations had been taken were calculated, and logarithmic differences formed exactly as had been done with the observations. These differences were now compared with those derived from observation, and from them was drawn the following list of the outstanding errors (C−O), expressed in units of the fourth place of decimals, where the sign + represents a greater absorption of radiant heat in the lower regions of the atmosphere at the time of observation than under average circumstances.

Comparison of observations with reducing curve in zenith-distance.

1871.	Zenith-distances.		C−O.	
Mar. 2.	61°·4	68°·7		−304
	”	73·0		−80
	”	76·6		−218
Mar. 27.	42·5	45·3		−355
	”	47·8	+56	
	”	51·7	+213	
	”	55·1	+260	
	”	62·2	+673	
	”	65·3	+887	
	”	71·4	+552	
	”	74·4	+359	
	”	77·5	+530	
Mar. 28.	43·3	46·1		−34
	”	59·9		−573
	”	62·5		−474
	”	64·9		−83
	”	67·6		−373
	”	73·0		−682
	”	75·2		−1228
Mar. 29.	39·0	41·4		−12
	”	43·5	+65	
	”	58·9	+259	
	”	61·8	+195	
	”	65·2		−143
	”	67·5		−352
	”	70·7	+116	
	”	74·3	+334	
	”	76·8	+543	
Apr. 3.	46·0	46·4		−121
	”	46·6	+24	
	”	48·2		−31
	”	53·9	+22	
	”	62·1		−539
	”	67·5		−31
	”	75·8	+286	

1871.	Zenith-distances.		C−O.	
Apr. 30.	45°·1	47°·8		−210
	”	54·2		−79
	”	67·1		−97
	”	73·4		−356
	”	78·8		−1137
May 3.	62·3	66·3	+18	
May 28.	59·8	65·2	+1	
	”	70·1		−178
	”	74·9	+240	
	”	77·7	+291	
	”	79·6	+173	
May 29.	57·0	60·2		−122
	”	64·1	+167	
	”	78·5	+604	
May 30.	68·3	76·6		−99
June 1.	69·7	71·0		−64
	”	73·3		−85
	”	79·4		−105
June 2.	73·8	74·2		−725
	”	74·3		−683
Nov. 1.	41·5	49·9	+275	
	”	52·8	+425	
	”	55·3	+76	
Dec. 19.	57·8	59·6		−97
	”	60·7		−50
Dec. 20.	49·5	49·7		−288
	”	49·9	+41	
	”	50·0		−7
	”	68·5		−301
	”	75·8		−612
	”	77·8		−634

TABLE (continued).

1871.	Zenith-distances.		C−O.	
Dec. 25.	30°·0	30°·7	+ 53	
	,,	32·7		− 48
	,,	36·1	+128	
	,,	39·8	+247	
	,,	44·0	+154	
	,,	48·3	+111	
Dec. 26.	29·3	47·8	+501	
	,,	56·2	+132	
Dec. 28.	31·1	33·3	+201	
	,,	36·5	+178	
	,,	37·2		−161
	,,	40·3		−201
	,,	45·4		−235
	,,	49·9		−467
1872. Jan. 2.	51·0	51·2	+405	
Jan. 5.	67·9	68·2	+177	
Jan. 18.	45·4	46·8		− 36
	,,	48·0	+127	
	,,	54·7		− 52
	,,	56·6	+ 23	
	,,	58·2	+242	
	,,	68·2		−339
Jan. 19.	42·9	44·6	+449	
Jan. 20.	59·8	62·0	+103	
Jan. 22.	50·1	54·2		−454
	,,	57·2		−846
Jan. 30.	62·2	63·3	+ 15	
	,,	64·8	+317	
	,,	66·3		−224
	,,	68·0		−323
	,,	69·8		− 98

1872.	Zenith-distances.		C−O.	
Feb. 16.	39°·4	41°·7	+394	
	,,	43·1	+665	
	,,	45·0	+531	
	,,	65·7	+383	
	,,	67·5	+461	
Feb. 20.	60·7	62·7	+198	
	,,	64·5	+258	
	,,	66·4	+164	
Feb. 24.	42·8	43·3	+ 2	
	,,	43·4		−266
	,,	44·0		− 37
	,,	44·8	+ 82	
	,,	45·5		−394
	,,	46·7		−486
	,,	47·7	+493	
Mar. 21.	36·3	37·4	+ 32	
	,,	39·8	+234	
Mar. 22.	42·9	44·1	+ 53	
	,,	45·3		− 36
	,,	48·2	+ 88	
	,,	54·3	+117	
	,,	56·3	+163	
Mar. 24.	51·5	53·5		− 90
	,,	54·5		−117
	,,	56·9		−147
	,,	59·3		−216
Apr. 15.	57·1	59·3	+493	
	,,	61·4	+208	
Apr. 18.	38·8	42·2	+209	
	,,	45·3	+296	
	,,	46·8	+294	
	,,	56·3	+ 62	
	,,	58·3	+236	
	,,	60·3		− 48
	,,	64·4		− 52
	,,	66·7	+282	

These errors were now treated in the ingenious way employed by SEIDEL (Untersuchungen, p. 41)—a Table was formed in which the zenith-distance was the argument for both horizontal and vertical columns; the errors were now entered at both points of intersection corresponding to the pairs of altitudes to which they belong.

When the lesser zenith-distance is the argument for the vertical column, the sign given in the Table was retained, in the other case the sign was changed; in this way all the errors come to stand on one side of the diagonal as they are in the Table, and on the other side with changed signs. The vertical and horizontal columns were now examined to see whether one particular sign predominated in any part. In fact the following corrections of the values of φz were found to be indicated:—

Zen.-dist.		Log correction.
29°	−	0·0100
35	−	·0049
40	−	·0008
45	+	·0027
50	+	·0050
55	+	·0054
60	+	·0032
65	−	·0011
70	−	·0041
75	−	·0041
79	−	·0082

The corrected curve was then found to intersect SEIDEL'S at 38°·2 and at 44°·0, while it was 0·0034 below it at 29°·0, and 0·0038 below it at 50°·0. On comparing the two curves for the whole interval between 29° and 50°, the new curve was found to be exactly 0·0010 below SEIDEL'S on an average; and since, as has already been said, our observations do not extend to within less than 29° of the zenith, this correction of +0·0010 was applied to the whole of the Table deduced from the new curve. At the same time a small systematic error which had arisen from an oversight in the Table for log (g) was allowed for; the error from this source might have been neglected, as it was only 0·0001 for $z=35°·5$, increasing to 0·0004 at 78°·5. These corrections being made, and the tabular quantities being slightly adjusted so as to make the differences run more regularly, the following definite Table for the extinction of the moon's radiant heat in the atmosphere was obtained, where, as before, the argument is the true zenith-distance, and ϕz is the logarithmic factor for reducing to the zenith.

Corrected Table for reducing to the zenith.

Zen.-dist.	ϕz.		Zen.-dist.	ϕz.		Zen.-dist.	ϕz.		Zen.-dist.	ϕz.	
29°	0·0046	10	42°	0·0227	20	55°	0·0575	34	68°	0·1348	101
30	0·0056	11	43	0·0247	21	56	0·0609	36	69	0·1449	108
31	0·0067	11	44	0·0268	22	57	0·0645	38	70	0·1557	116
32	0·0078	12	45	0·0290	24	58	0·0683	42	71	0·1673	124
33	0·0090	12	46	0·0314	25	59	0·0725	46	72	0·1797	133
34	0·0102	13	47	0·0339	26	60	0·0771	52	73	0·1930	144
35	0·0115	14	48	0·0365	27	61	0·0823	58	74	0·2074	158
36	0·0129	14	49	0·0392	28	62	0·0881	64	75	0·2232	175
37	0·0143	15	50	0·0420	29	63	0·0945	69	76	0·2407	196
38	0·0158	16	51	0·0449	30	64	0·1014	74	77	0·2603	220
39	0·0174	17	52	0·0479	31	65	0·1088	79	78	0·2823	246
40	0·0191	18	53	0·0510	32	66	0·1167	87	79	0·3069	
41	0·0209	18	54	0·0542	33	67	0·1254	94			
42	0·0227		55	0·0575		68	0·1348				

A comparison with SEIDEL's Table* given below at once shows that the extinction of the moon's radiant heat, although following a very similar law to that for stellar light, is not so great at considerable zenith-distances.

At 79° zenith-distance the difference is about one tenth of the whole amount†.

On the Connexion between the Moon's Phase and the amount of her Radiant Heat.

The observations were now reduced to the zenith, and the mean zenith-values deduced from each night, each observation having a weight apportioned to it inversely proportional to the number of which ϕz is the logarithm. With the zenith-means and the extinction Table the values in column "G calc." of the following Table were calculated, and from the differences between observation and calculation the probable error of each observation and of the zenith-mean, z, was deduced by the usual formulæ. These are given in columns *p.e.* and $p.e_z$.

The probable errors are of course based on the assumption that no constant source of error existed; we shall, however, see further on that most probably large constant errors were present on many occasions.

* For the convenience of English readers we give SEIDEL's Table for the extinction of light in the atmosphere (Untersuchungen, p. 43) (for full title, see note, p. 595).

For $z < 14°$ the values of ϕz are imperceptible.

z.	ϕz.	z.	ϕz.	z.	ϕz.	z.	ϕz.		z.	ϕz.	
13°	0·000	28°	0·006	43°	0·023	58°	0·083	7	73°	0·233	16
14	0·001	29	0·007	44	0·026	59	0·090	7	74	0·249	19
15	0·001	30	0·007	45	0·028	60	0·097	7	75	0·268	20
16	0·001	31	0·008	46	0·031	61	0·104	8	76	0·288	21
17	0·001	32	0·009	47	0·034	62	0·112	9	77	0·309	24
18	0·002	33	0·010	48	0·038	63	0·121	9	78	0·333	26
19	0·003	34	0·011	49	0·041	64	0·130	10	79	0·359	29
20	0·003	35	0·012	50	0·045	65	0·140	10	80	0·388	40
21	0·003	36	0·013	51	0·049	66	0·150	10	81	0·428	56
22	0·003	37	0·014	52	0·053	67	0·160	10	82	0·484	65
23	0·004	38	0·015	53	0·057	68	0·170	10	83	0·549	67
24	0·004	39	0·016	54	0·062	69	0·180	11	84	0·616	68
25	0·005	40	0·017	55	0·067	70	0·191	13	85	0·684	70
26	0·005	41	0·019	56	0·072	71	0·204	14	86	0·754	
27	0·006	42	0·021	57	0·077	72	0·218	15			
28	0·006	43	0·023	58	0·083	73	0·233				

At p. 503 of the 'Abhandlungen der Math.-phys. Classe der K. Akademie der Wissenschaften,' Munich, 1861, Professor SEIDEL gives a new Table of ϕz based on seventeen years' observations; but it differs so little (at most 1 per cent.) from the Table here given, that Professor SEIDEL himself, for the sake of uniformity, continued to make use of the original Table only. Where the difference between the Tables for light and heat is greatest, Professor SEIDEL's new Table lessens this difference by about one seventh part.

† *Note added Dec.* 27, 1873.—Since writing the above, a possible way of accounting for the difference between the heat- and light-extinction curves has been found in the circumstance that the former was obtained from observations on the moon, the latter from observations on the stars. In the former case less loss would arise from bad definition at low altitudes than in the latter, as the greater part of the light and heat would be simply transferred from one part of the moon's image to another; whereas in the latter case it would be transferred from the star's image to the sky round it, when it would only tend to lessen the contrast. Consequently it does not necessarily follow that the foregoing observations point to different laws for light and heat.

Reduction of observations to the zenith.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1871.	Zenith-distance.	Observed G corr.	G calc.	C – O.	*p.e.*	Zenith-mean (*z*).	*p.e$_z$.*
Mar. 2.	61·4°	93·1	95·1	+ 2·0	± 1·5	115·6	±1·1
	68·7	86·3	83·4	– 2·9			
	73·0	72·9	74·1	+ 1·2			
	76·6	65·4	64·6	– 0·8			
Mar. 27.	42·5	34·9	33·3	– 1·6	± 1·7	35·13	±0·68
	45·3	37·6	32·8	– 4·8			
	47·8	33·8	32·3	– 1·5			
	51·7	31·8	31·5	– 0·3			
	55·1	31·4	30·8	– 0·6			
	62·2	25·8	28·6	+ 2·8			
	65·3	23·2	27·2	+ 4·0			
	71·4	21·6	23·6	+ 2·0			
	74·4	20·5	21·5	+ 1·0			
	77·5	17·2	18·8	+ 1·6			
Mar. 28.	43·3	41·4	44·8	+ 3·4	± 2·1	47·47	±0·95
	46·1	41·3	44·1	+ 2·8			
	59·9	42·1	39·8	– 2·3			
	62·5	39·6	38·5	– 1·1			
	64·9	34·7	37·0	+ 2·3			
	67·6	35·0	35·1	+ 0·1			
	73·0	32·5	30·4	– 2·1			
	75·2	34·1	28·2	– 5·9			
Mar. 29.	39·0	67·0	66·1	– 0·9	± 1·8	68·79	±0·72
	41·4	66·7	65·4	– 1·3			
	43·5	65·1	64·8	– 0·3			
	58·9	56·3	58·3	+ 2·0			
	61·8	54·9	56·3	+ 1·4			
	65·2	55·8	53·4	– 2·4			
	67·5	55·8	51·0	– 4·8			
	70·7	46·2	47·2	+ 1·0			
	74·3	39·4	42·2	+ 2·8			
	76·8	33·7	38·2	+ 4·5			
Apr. 3.	46·6	314·6	320·5	+ 5·9	± 9·8	345·7	±4·1
*	46·0	317·1	321·6	+ 4·5			
	46·4	325·4	320·8	– 4·6			
	48·2	316·0	317·5	+ 1·5			
	53·9	301·0	305·4	+ 4·4			
	62·1	313·3	281·8	–31·5			
	67·5	250·6	256·3	+ 5·7			
	75·8	181·6	200·3	+18·7			
Apr. 30.	45·1	204·8	213·1	+ 8·3	± 7·9	228·0	±4·1
	47·8	212·2	209·9	– 2·3			
	54·2	197·2	200·9	+ 3·7			
	67·1	164·3	170·4	+ 6·1			
	73·4	147·1	144·3	– 2·8			
	78·8	137·3	113·8	–23·5			
May 3.	66·3	298·1	299·9	+ 1·8	± 1·9	395·0	±1·7
	62·3	322·9	320·8	– 2·1			

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1871.	Zenith-distance.	Observed G corr.	G calc.	C−O.	*p.e.*	Zenith-mean (*z*).	*p.ez.*
May 28.	59°·8	88·5	86·5	− 2·0	± 2·0	103·1	± 1·2
	65·2	80·9	80·0	− 0·9			
	70·1	75·4	71·9	− 3·5			
	74·9	58·9	61·9	+ 3·0			
	77·7	51·1	54·6	+ 3·5			
	79·6	46·8	49·1	+ 2·3			
May 29.	57·0	121·9	119·3	− 2·6	± 4·5	138·4	± 3·0
	60·2	121·2	115·6	− 5·6			
	64·1	106·6	109·4	+ 2·8			
	78·5	60·9	70·3	+ 9·4			
May 30.	68·3	138·5	139·4	+ 0·9	± 1·1	191·4	± 1·1
	76·6	108·4	107·2	− 1·2			
June 1.	69·7	278·3	281·6	+ 3·3	± 1·6	400·0	± 1·3
	71·0	273·0	272·2	− 0·8			
	73·3	256·4	254·0	− 2·4			
	79·4	193·1	192·7	− 0·4			
June 2.	74·3	326·5	311·5	−15·0	±19·4	507·5	±18·1
*	73·8	284·1	317·0	+32·9			
	74·2	330·9	312·5	−18·4			
Nov. 1.	55·3	139·2	136·0	− 3·2	± 3·7	155·6	± 2·0
	52·8	130·8	138·6	+ 7·8			
	49·9	141·3	141·4	+ 0·1			
	41·5	152·4	148·0	− 4·4			
Dec. 19.	57·8	87·5	84·4	− 3·1	± 1·9	98·58	± 1·28
	59·6	81·8	82·9	+ 1·1			
	60·7	79·8	81·9	+ 2·1			
Dec. 20.	49·9	123·3	128·1	+ 4·8	± 3·2	141·0	± 1·5
	49·5	124·7	128·4	+ 3·7			
	49·7	133·1	128·2	− 4·9			
	50·0	124·6	128·0	+ 3·4			
	68·5	102·6	102·2	− 0·4			
	75·8	87·4	81·7	− 5·7			
	77·8	79·5	74·4	− 5·1			
Dec. 25.	48·3	356·1	351·4	− 4·7	± 4·3	382·9	± 1·7
	44·0	359·1	359·9	+ 0·8			
	39·8	355·8	366·6	+10·8			
	36·1	367·8	371·5	+ 3·7			
	32·7	384·3	375·3	− 9·0			
	30·7	376·1	377·2	+ 1·1			
	30·0	380·8	377·9	− 2·9			
Dec. 26.	56·2	315·4	306·3	− 9·1	±10·6	353·1	± 6·6
*	47·8	306·8	324·9	+18·1			
	29·3	358·0	349·0	− 9·0			

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1871.	Zenith-distance.	Observed G corr.	G calc.	C−O.	*p.e.*	Zenith-mean (*z*).	*p.e$_z$.*
Dec. 28.	49°·9	305·2	275·5	−29·7	±12·5	303·3	±5·0
	45·4	295·6	283·0	−12·6			
	40·3	298·2	289·9	− 8·3			
	37·2	297·1	293·3	− 3·8			
	31·1	288·1	298·6	+10·5			
	33·3	274·8	296·8	+22·0			
	36·5	275·2	294·0	+18·8			
1872.							
Jan. 2.	51·0	93·6	89·4	− 4·2	± 4·1	99·2	±3·1
	51·2	85·2	89·3	+ 4·1			
Jan. 5.	68·2	17·6	18·0	+ 0·4	± 0·4	24·67	±0·41
	67·9	18·5	18·1	− 0·4			
Jan. 18.	45·4	103·4	103·4	0·0	± 2·1	110·7	±0·9
*	46·8	103·6	102·6	− 1·0			
*	48·0	99·3	101·8	+ 2·5			
	54·7	99·0	97·2	− 1·8			
	56·6	95·7	95·8	+ 0·1			
*	58·2	89·7	94·4	+ 4·7			
	68·2	86·1	80·8	− 5·3			
Jan. 19.	42·9	136·8	131·7	− 5·1	± 4·9	139·3	±3·7
	44·6	125·4	130·6	+ 5·2			
Jan. 20.	59·8	185·2	182·7	− 2·5	± 2·5	217·8	±2·0
	62·0	175·3	177·8	+ 2·5			
Jan. 22.	50·1	163·3	180·5	+17·2	±11·5	199·0	±7·5
	54·2	176·4	175·4	− 1·0			
	57·2	188·2	171·2	−17·0			
Jan. 27.	70·4	247·0	......	......	......	357·2	
Jan. 30.	69·8	116·2	115·1	− 1·1	± 4·2	163·9	±2·2
	68·0	127·9	120·1	− 7·8			
	66·3	129·9	124·6	− 5·3			
*	64·8	118·4	128·0	+ 9·6			
	63·3	130·5	131·2	+ 0·7			
*	62·2	130·4	133·5	+ 3·1			
Feb. 1.	66·1	70·7	......	......	......	92·6	
Feb. 16.	39·4	88·8	81·3	− 7·5	± 2·8	84·8	±1·3
	41·7	80·7	80·6	− 0·1			
	43·1	75·5	80·1	+ 4·6			
	45·0	77·4	79·3	+ 1·9			
	65·7	65·1	65·2	+ 0·1			
	67·5	61·4	62·8	+ 1·4			
Feb. 20.	60·7	197·4	189·8	− 7·6	± 3·6	228·4	±2·3
	62·7	182·9	184·6	+ 1·7			
	64·5	174·6	179·4	+ 4·8			
	66·4	171·5	173·3	+ 1·8			

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1872.	Zenith-distance.	Observed G corr.	G calc.	C−O.	*p.e.*	Zenith-mean (*z*).	*p.e.z.*
Feb. 24.	46°·7	345·2	313·5	−31·7	±14·9	338·5	±5·6
	45·5	339·2	315·7	−23·5			
	43·4	331·8	319·2	−12·6			
	42·8	312·8	320·0	+ 7·2			
	43·3	312·3	319·3	+ 7·0			
	44·0	314·5	318·2	+ 3·7			
	44·8	304·9	316·9	+12·0			
	47·7	273·9	311·8	+37·9			
Mar. 21.	36·3	298·3	293·2	− 5·1	± 5·1	302·3	±3·0
	37·4	295·5	292·0	− 3·5			
	39·8	280·8	289·5	+ 8·7			
Mar. 22.	42·9	289·2	286·7	− 2·5	± 2·6	303·4	±1·2
	44·1	284·7	285·1	+ 0·4			
	45·3	289·3	283·3	− 6·0			
	48·2	277·4	278·6	+ 1·2			
	54·3	264·9	267·2	+ 2·3			
	56·3	258·0	263·0	+ 5·0			
Mar. 24.	59·3	353·1	345·7	− 7·4	± 4·2	409·8	±2·1
	51·5	359·1	368·3	+ 9·2			
	53·5	361·6	363·1	+ 1·5			
	54·5	361·2	360·4	− 0·8			
	56·9	356·6	353·6	− 3·0			
Mar. 26.	68·1	250·7	......	......	......	342·7	
Apr. 15.	57·1	57·8	54·7	− 3·1	± 2·1	63·52	±1·46
	59·3	50·4	53·6	+ 3·2			
	61·4	52·3	52·3	0·0			
Apr. 18.	38·8	175·8	172·2	− 3·6	± 3·3	179·1	±1·3
	42·2	166·1	169·8	+ 3·7			
	45·3	161·1	167·3	+ 6·2			
	46·8	163·9	165·9	+ 2·0			
	56·3	158·8	155·2	− 3·6			
	58·3	153·1	152·7	− 0·4			
	60·3	155·9	149·5	− 6·4			
	64·4	145·9	140·8	− 5·1			
	66·7	128·7	135·0	+ 6·3			

Note.—The notes, which are the same as for the unreduced observations, will be found at pp. 589–592.

The observations of each night are now represented by a single quantity, so that, after allowing for the varying distance of the moon from the earth and sun, they will be available for determining the rate of change of the moon's radiant heat with her phase. In the following Table the values of log z* are arranged in order of ε. Column I. gives the astronomical date, column II. the apparent elongation from full

* See column VII. of the foregoing Table.

moon (— before full moon, + after), column III. the log z; column IV. contains twice the logarithm of the moon's radius-vector (ϱ) from the earth, the radius-vector of the moon when her horizontal parallax is 57′ being taken as unity; column V. contains twice the logarithm of the moon's distance (R′) from the sun*, the mean distance of the earth being taken as unity.

The natural number, z', corresponding to the sum of the quantities given in columns III., IV., and V., or the observed mean heating-effect on each night reduced to the zenith, is given in column VI.; in VII. is given w, the number of sets of observations, each set consisting of ten determinations, from which z' has been deduced; and in IX. the difference between columns VI and VIII.

Reduction of daily means to a mean distance of the Sun and Moon, and comparison with Curve. (See also Plate XLVIII., A.)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
1800+	ε.	log z.	log ϱ^2.	log R'^2.	z' observed.	w=No. of sets.	z' curve.	C−O.	$\frac{100}{z'}$(C−O).
Mar. 24, 71.	−138° 0′	....	....	....	0·0	2	......	......	......
Mar. 27, 71.	−105 19	1·5457	0·0356	9·9982	38·0	10	38·0	0·0	0·0
Mar. 28, 71.	− 94 0	1·6764	0·0307	9·9988	50·8	8	54·2	+ 3·4	+ 6·3
Apr. 15, 72.	− 89 47	1·8029	0·0352	0·0037	69·5	3	63·8	− 5·7	− 8·9
Feb. 16, 72.	− 83 12	1·9284	0·0283	9·9902	88·5	6	80·7	− 7·8	− 9·7
Mar. 29, 71.	− 82 33	1·8376	0·0229	9·9995	72·4	10	82·3	+ 9·9	+12·0
Dec. 19, 71.	− 78 20	1·9938	0·0029	9·9862	96·1	3	93·7	− 2·4	− 2·6
Jan. 18, 72.	− 74 11	2·0443	0·0221	9·9866	113·0	7	105·1	− 7·9	− 7·5
May 28, 71.	− 72 14	2·0134	9·9800	0·0126	101·4	6	110·5	+ 9·1	+ 8·2
Dec. 20, 71.	− 66 8	2·1493	0·0151	9·9865	141·6	7	128·8	−12·8	− 9·9
Jan. 19, 72.	− 63 7½	2·1441	0·0275	9·9870	144·1	2	138·5	− 5·6	− 4·0
May 29, 71.	− 59 55	2·1412	9·9661	0·0131	131·9	4	149·3	+17·4	+11·6
Apr. 18, 72.	− 56 11	2·2531	0·0164	0·0057	188·5	9	162·5	−26·0	−16·0
Apr. 30, 71.	− 53 46	2·3579	9·9753	0·0082	219·5	6	171·5	−48·0	−28·0
Jan. 20, 72.	− 50 56	2·3380	0·0366	9·9876	230·2	2	182·3	−47·9	−26·3
Mar. 2, 71.	− 49 36	2·0628	0·0145	9·9943	118·0	4	187·6	+69·6	+37·1
May 30, 71.	− 45 37	2·2821	9·9527	0·0138	177·2	2	203·8	+26·6	+13·0
Feb. 20, 72.	− 38 9	2·3588	0·0365	9·9924	244·1	4	236·3	− 7·8	− 3·3
Mar. 21, 72.	− 36 42	2·4804	0·0173	9·9993	314·1	3	243·0	−71·1	−29·3
Jan. 22, 72.	− 28 44	2·2988	0·0388	9·9883	211·8	3	282·6	+70·8	+25·0
Mar. 22, 72.	− 24 27	2·4820	0·0100	9·9998	310·4	6	307·0	− 3·4	− 1·1
Apr. 3, 71.	− 21 31	2·5387	9·9628	0·0025	319·2	8	325·5	+ 6·3	+ 1·9
June 1, 71.	− 17 29	2·6021	9·9346	0·0145	355·8	4	351·4	− 4·4	− 1·3
May 3, 71.	− 14 16	2·5966	9·9397	0·0097	351·6	2	371·4	+19·8	+ 5·3
Dec. 25, 71.	− 10 16	2·5830	0·0366	9·9876	404·8	7	393·1	−11·7	− 3·0
Mar. 24, 72.	− 4 9	2·6126	9·9940	0·0005	404·7	5	407·2	+ 2·5	+ 0·6
June 2, 71.	− 3 42	2·7054	9·9312	0·0148	448·1	3	407·2	−40·9	−10·0
Dec. 26, 71.	+ 0 45	2·5478	0·0385	9·9877	375·0	3	401·4	+26·4	+ 6·6
Feb. 24, 72.	+ 8 33	2·5295	0·0107	9·9936	341·8	8	373·1	+31·3	+ 8·4
Dec. 28, 71.	+ 23 28	2·4819	0·0337	9·9874	318·4	7	307·5	−10·9	− 3·5
Mar. 26, 72.	+ 24 44	2·5349	9·9806	0·0008	328·3	1	301·8	−26·5	− 8·8
Jan. 27, 72.	+ 25 19	2·5528	0·0306	9·9888	373·5	1	299·2	−74·3	−24·8
Nov. 1, 71.	+ 52 2	2·1920	0·0363	9·9943	166·9	4	182·9	+16·0	+ 8·7
Jan. 30, 72.	+ 61 53	2·2146	0·0003	9·9882	159·6	6	149·0	−10·6	− 7·1
Jan. 2, 72.	+ 82 30	1·9965	9·9963	9·9856	95·2	2	96·5	+ 1·3	+ 1·3
Feb. 1, 72.	+ 87 54	1·9668	9·9778	9·9876	85·6	1	83·1	− 2·5	− 3·0
Jan. 5, 72.	+119 51	1·3922	9·9598	9·9843	21·7	2	21·7	0·0	0·0

* The linear distance of the moon from the sun is readily obtained from the earth's radius-vector by the formula

$$\text{Moon's distance from sun} = R' = R + \frac{8''{\cdot}95}{P} \cos \varepsilon,$$

where R is the earth's radius-vector, P the moon's equatorial horizontal parallax, and ε the moon's elongation from full; in our ase we have adopted a mean value of P=3420″, so that R′=R+[7·4178] cos ε nearly.

The values of z' were now laid down as ordinates with those of ε as abscissæ, the weight being indicated on the plan previously mentioned. A curve was then carefully drawn and read off and then compared with the observations; in this way small corrections for the curve were deduced, and finally the following Table adopted as giving the best representation of the observations. (See also Plate XLVIII., A.)

Phase Table.

ε.	z'.		ε.	z'.		ε.	z'.		ε.	z'.	
−106°	37·5	1·5	−48°	194·0	8·2	+10°	366·9	8·8	+ 68°	132·5	5·1
104	39·0	2·1	46	202·2	8·4	12	358·1	8·8	70	127·4	5·0
102	41·1	2·6	44	210·6	8·6	14	349·3	8·8	72	122·4	5·0
100	43·7	3·0	42	219·2	8·8	16	340·5	8·8	74	117·4	4·9
98	46·7	3·6	40	228·0	9·0	18	331·7	8·9	76	112·5	4·9
96	50·3	3·9	38	237·0	9·3	20	322·8	8·8	78	107·6	4·9
94	54·2	4·4	36	246·3	9·6	22	314·0	8·9	80	102·7	5·0
92	58·6	4·7	34	255·9	9·9	24	305·1	8·9	82	97·7	4·9
90	63·3	4·8	32	265·8	10·2	26	296·2	8·9	84	92·8	5·0
88	68·1	5·1	30	276·0	10·6	28	287·3	8·9	86	87·8	4·9
86	73·2	5·3	28	286·6	11·2	30	278·4	8·9	88	82·9	4·9
84	78·5	5·3	26	297·8	11·9	32	269·5	8·8	90	78·0	4·9
82	83·8	5·4	24	309·7	12·7	34	260·7	8·9	92	73·1	4·8
80	89·2	5·4	22	322·4	12·9	36	251·8	8·9	94	68·3	4·8
78	94·6	5·5	20	335·3	12·9	38	242·9	8·8	96	63·5	4·7
76	100·1	5·5	18	348·2	12·7	40	234·1	8·9	98	58·8	4·5
74	105·6	5·6	16	360·9	12·1	42	225·2	8·9	100	54·3	4·3
72	111·2	5·8	14	373·0	11·3	44	216·3	8·7	102	50·0	4·1
70	117·0	6·0	12	384·3	10·0	46	207·6	8·6	104	45·9	3·9
68	123·0	6·2	10	394·3	7·7	48	199·0	8·2	106	42·0	3·7
66	129·2	6·4	8	402·0	4·2	50	190·8	7·8	108	38·3	3·5
64	135·6	6·6	6	406·2	+ 1·1	52	183·0	7·6	110	34·8	3·3
62	142·2	6·8	4	407·3	− 1·0	54	175·4	7·4	112	31·5	3·0
60	149·0	7·0	− 2	406·3	3·0	56	168·0	6·9	114	28·5	2·6
58	156·0	7·2	0	403·3	5·0	58	161·1	6·4	116	25·9	2·3
56	163·2	7·4	+ 2	398·3	6·7	60	154·7	6·1	118	23·6	2·0
54	170·6	7·6	4	391·6	7·9	62	148·6	5·6	+120	21·6	
52	178·2	7·8	6	383·7	8·3	64	143·0	5·3			
50	186·0	8·0	8	375·4	8·5	66	137·7	5·2			
− 48	194·0		+10	366·9		+68	132·5				

From this Table the numbers in column VIII. were taken out. Column IX. shows the difference between curve and observation; and for the purpose of giving a better idea of the degree of agreement, the percentage error is given in column X. The sum of $w\left\{\frac{100}{z'}(\mathrm{C}-\mathrm{O})\right\}^2$ is 25917·4, and the number of nights, omitting the first two and the last, is 34; whence the mean percentage error of a single set of ten readings has been calculated on the usual assumption that the errors are inversely proportional to the square root of the number of sets.

This mean error is $\pm 28{\cdot}02$ per cent., corresponding to a probable error of $\pm 18{\cdot}9$ per cent. The average number of sets is 4·7; this gives $\pm 12{\cdot}9$ per cent. for the mean and $\pm 8{\cdot}7$ per cent. for the probable error of a night's observations; a comparison of these numbers with column IX., however, shows that large constant errors were doubtless prevalent on many nights, so that any increase of the number of sets was almost powerless to obtain a more reliable result. Irrespective of the number of sets and excluding, as a matter of course, the first and last determinations, the mean error is $\pm 10 \cdot 4$ per cent.

For the sake of comparison we may state that SEIDEL found the probable error of a single complete determination (consisting of four readings of his photometer) of the relative brightness of two stars of the first magnitude to be $\frac{1}{12}$, or 8·3 per cent. By inspection of the curve it will be perceived that the maximum of heat appears to take place a little before full moon, but that the heat at the time of the first quarter is rather less than at the last quarter; and also that the average heat during the period from first quarter to full moon is less than during the corresponding period after full. Both these departures of the curve from symmetry on the two sides of the full appear somewhat too large to be ascribed entirely to accidental error in the observations. Possibly the considerable inequality in the distribution of the mountainous regions and the plains and so-called "seas" between the preceding and following halves of the moon's visible surface may be looked to as the cause; and it may be desirable to direct attention to this point in a future course of observations, but for the present it would evidently be premature to speculate further. As to the amount of the difference, it is found by examination of the curve that at the first quarter the amount is 63·3, and at the last quarter 78·0, that just before full moon being 407·3.

In conjunction with the foregoing experiments determinations of the proportion of the moon's heat transmitted by a plate of glass were occasionally made. The following is a Table corresponding to those at pp. 589–592 and 600–603 for the moon's unobstructed heat.

Journal and Reduction to zenith of observations made with glass plate interposed.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
1871.	Sidereal time.	Mean deviation (G').	Zenith-distance.	Observed G' corr.	G' calc.	Difference of columns V. and VI. C–O.	*p.e.*	Zenith-mean (*z*).	*p.ez.*
	h m		°						
Apr. 3.	11 11	42·2	46·2	42·2	45·3	+3·1	±2·5	48·7	±1·2
	11 48	42·6	46·0	42·6	45·3	+2·7			
	12 27	47·6	47·1	47·6	45·1	–2·5			
	13 33	40·9	51·8	40·9	43·7	+2·8			
	15 23	40·1	64·7	40·1	38·2	–1·9			
	16 13	38·8	71·4	38·8	32·8	–6·0			
Apr. 30.	12 34	24·8	46·4	24·8	25·0	+0·2	±1·9	26·9	±1·1
	13 13	24·8	49·8	24·8	24·4	–0·4			
	14 16	18·6	56·7	18·6	23·2	+4·6			
	15 52	20·7	69·9	20·7	18·8	–1·9			
	16 34	18·2	75·8	18·2	15·6	–2·6			
May 3.	12 21	41·4	63·6	41·4	43·2	+1·8	±1·7	54·2	±1·5
	13 56	46·7	61·0	46·5	44·8	–1·7			
May 28.	15 10	8·5	62·4	8·5	8·5	0·0	±0·2	10·4	±0·2
	15 50	7·9	67·8	7·9	7·7	–0·2			
	16 23	6·4	72·5	6·4	6·8	+0·4			
May 30.	16 42	22·3	70·6	22·3	......	......	.....	32·4	
June 1.	16 13	42·0	70·2	42·0	43·8	+1·8	±2·1	63·1	±1·9
	16 52	40·2	72·0	40·2	41·7	+1·5			
	17 30	42·2	74·5	42·2	38·4	–3·8			
June 2.	16 8	63·6	73·9	63·6	63·5	–0·1	±0·1	102·0	±0·1
	16 48	63·1	74·0	63·1	63·2	+0·1			
Dec. 20.	2 22	12·7	50·5	12·7	......	......	......	14·0	
Dec. 25.	2 25	52·1	46·0	52·3	51·7	–0·6	±0·5	55·5	±0·3
	2 56	51·6	41·9	51·7	52·7	+1·0			
	3 31	54·3	37·8	54·3	53·6	–0·7			
	4 45	54·5	31·5	54·3	54·6	+0·3			
Dec. 28.	4 53	40·4	47·5	40·4	34·0	–6·4	±3·0	36·8	±1·6
	5 31	34·5	42·6	34·4	34·9	+0·5			
	9 9	34·0	32·1	34·1	36·2	+2·1			
	9 45	32·2	34·8	32·3	35·9	+3·6			
1872. Feb. 24.	12 37	41·5	46·2	41·5	41·1	–0·4	±0·4	44·3	±0·3
	13 15	39·8	49·8	39·9	40·3	+0·4			
Mar. 21.	10 27	34·3	36·7	34·3	31·8	–2·5	±2·5	32·9	±1·7
	10 59	29·2	38·2	29·2	31·7	+2·5			
Mar. 22.	12 40	32·7	46·7	32·7	32·4	–0·3	±0·4	34·9	±0·3
	13 13	31·0	50·0	31·0	31·7	+0·7			
	14 21	30·1	58·5	30·1	29·7	–0·4			
Mar. 24.	14 5	48·8	55·5	48·8	......	.....	......	55·9	
Apr. 18.	11 14	16·7	39·7	16·8	16·3	–0·5	±0·5	17·0	±0·4
	12 9	15·5	43·6	15·5	16·0	+0·5			

The corrections for the changes of the moon's phase and semidiameter during the observations were very small, on which account the particulars are omitted, as also the adopted mean elongation, semidiameter, and sidereal time, which are the same as for the total heat determinations. The observations are satisfactorily represented in altitude by the Table already deduced. Although these results were fully examined by means of a curve like that for the total heat, it is not necessary to give further particulars here, as these observations will afterwards be combined with others, of which we must now give an account.

Observations made during the Autumn of 1870.

The preceding investigations having thus given, at least, a tolerably close approximation to the law of the extinction of the moon's radiant heat in our atmosphere, and the extent to which its amount is influenced by her change of phase, we are in a position to make use of a compact series of observations made in the autumn of 1870. These are, as a rule, confined to low altitudes; and the range on any given night is, with one or two exceptions, so small that they could not have been used with advantage in the construction of the altitude curve, nor, owing to alterations in the height of the directive magnet of the galvanometer, were they at once available for the determination of, or comparison with, the phase curve. However, as they were made for the most part under very favourable circumstances of weather, and as from their chiefly referring to the waning moon they are important as filling an all but vacant gap in the records of 1871–72, they are introduced here, although out of their chronological order. The particulars of the observations are arranged in nearly the same way as those already discussed, the only difference being that the sets taken with a plate of glass interposed are left in their original place in the journal. As a matter of convenience, and on account of its greater accordance with fact, the Phase Table at p. 605 was used in place of LAMBERT'S formula for the correction for the change of phase. The Table of ϕz was used for reducing to the zenith. In this way the quantities in column "log (G corr.)" were obtained.

Occasionally the time of observation was not noted, but it could generally be determined with sufficient accuracy from the measured altitudes of the moon. As has been already mentioned, some uncertainty exists as to the height at which the directive magnet was placed on some nights; but as it was always used in one or other of two positions (3·00 or 3·46 inches), the directive force on the needle being as little as 0·6579 in one position as compared with unity in the other, little anxiety can arise as to the correctness of the assumptions made.

The following Tables contain the Journal of Observations and their reductions, which will be readily understood on comparison with those already given.

Journal of Observations and Preliminary Reductions.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
			The Moon's							The adopted mean		
1870.	Sidereal time.	Observed deviation (G.).	ε.	Apparent zenith-distance.	Apparent semi-diameter.	log (*g*).	log (ε).	log (σ).	log (G corr.).	Sidereal time.	ε.	Apparent semi-diameter.
	h m		° ′	°	′ ″					h m	° ′	′ ″
Aug. 8.	19 13	97·8	−30 33	76·4	15 56·1	9·9998	0·0019	0·0000	1·9920	19 45·0	−30 19	15 56·0
	19 31	83·0	−30 25	76·5	15 56·1	9·9999	0·0008	0·0000	1·9198			
*	20 1	77·4	−30 13	77·0	15 56·0	9·9999	9·9991	0·0000	1·8877			
† *gl*	20 13	11·0	−30 8	77·4	15 55·9	0·0000	9·9984	0·0000	1·0398			
Aug. 10‡.	20 29	{ 162·7 / *gl* 36·0 }	− 5 59	73·3	15 40·0	{ 9·9995 / 0·0000 }	0·0000	0·0000	2·2109	20 29·0	− 5 59	15 40·0
Aug. 11.	21 18	173·4	+ 8 29	70·0	15 30·8	9 9995	9·9992	0·0000	2·2377	21 50·0	+ 8 39	15 30·8
§ *gl*	21 33	34·7	+ 8 34	69·8	15 30·8	0·0000	9·9996	0·0000	1·5399			
	21 48	166·6	+ 8 38	69·6	15 30·8	9·9995	9·9999	0·0000	2·2211			
gl	22 3	32·3	+ 8 43	69·5	15 30·9	0·0000	0·0003	0·0000	1·5095			
	22 28	155·9	+ 8 51	69·7	15 30·9	9·9996	0·0010	0·0000	2·1934			
Aug. 12.	19 55	133·6	+19 35	75·8	15 19·6	9·9997	9·9956	0·0022	2·1233	22 0·0	+20 19	15 21·9
‖	21 35	172·6	+20 10	67·5	15 21·6	9·9995	9·9991	0·0002	2·2358			
¶ *gl*	21 52	27·9	+20 16	66·8	15 21·8	0·0000	9·9997	0·0002	1·4455			
	22 9	181·3	+20 22	66·2	15 22·0	9·9994	0·0003	0·0000	2·2581			
gl	22 25	32·0	+20 27	65·8	15 22·0	0·0000	0·0008	0·0000	1·5059			
	22 40	181·6	+20 32	65·5	15 22·1	9·9994	0·0013	0·0000	2·2598			
Aug. 13.	19 49	74·7	+31 0	78·2	15 10·2	9·9999	9·9954	0·0028	1·8714	20 30·0	+31 38	15 12·9
** *gl*	20 15	15·1	+31 10	75·0	15 11·0	0·0000	9·9966	0·0020	1·7776			
	20 27	102·8	+31 15	73·6	15 11·4	9·9998	9·9971	0·0016	2·0105			
gl	20 45	16·5	+31 22	71·6	15 11·8	0·0000	9·9980	0·0012	1·2167			
	21 4	124·7	+31 28	69·7	15 12·3	9·9997	9·9987	0·0008	2·0951			
	22 49	145·2	+32 4	62·0	15 14·1	9·9996	0·0029	9·9992	2·1637			
gl	23 4	21·0	+32 8	61·5	15 14·3	0·0000	0·0034	9·9990	1·3246			
	23 19	148·9	+32 12	61·1	15 14·3	9·9996	0·0038	9·9990	2·1753			
Aug. 14.	21 6	146·7	+42 49	70·3	15 4·2	9·9996	9·9982	0 0008	2·1651	21 40·0	+43 2	15 5·0
†† *gl*	21 21	14·5	+42 55	68·6	15 4·6	0·0000	9·9990	0·0004	1·1608			
	21 37	147·2	+43 0	66·9	15 5·0	9·9996	9·9997	0·0000	2·1672			
gl	21 51	15·4	+43 6	65·4	15 5·3	0·0000	0·0006	9·9998	1·1879			
	22 16	142·0	+43 14	62·9	15 6·1	9·9996	0·0018	9·9988	2·1525			

* Aug. 8.—No mention of cloud. At 20^h 1^m the directive magnet was at 3·46 in.; the reading has already been reduced to what it would have been with the magnet at 3·00 in. See preceding page.

† 20^h 13^m. This and all subsequent experiments marked *gl* were made with the glass plate interposed.

‡ Aug. 10.—Sky more or less streaky all night. The only available observations on this night were three sets, giving 252·1, *gl* 54·7, and 242·5 with the magnet at 3·46 in., for which the sidereal time 20^h 29^m has been adopted as best suiting the recorded altitudes.

§ Aug. 11.—The original readings were taken with the magnet at 3·46 in.; the proper reduction to 3·00 in. has been made. Sky slightly streaky all this night.

‖ Aug. 12.—There is no mention of the height of the magnet on this and the following night; but as the observations were made with it at 3·46 in. on the 11th and 14th, it is assumed to have been at the same height on the 12th and 13th.

¶ The telescope not on the moon for about half a minute.

** Aug. 13.—Magnet at 3·46 in., observations reduced to 3·00 in.

†† Aug. 14.—E.N.E. wind all night; galvanometer very unsteady. The magnet mentioned as being at 3·46 in.; observations reduced to 3·00 in.

TABLE (continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
			The Moon's							The adopted mean		
1870.	Sidereal time.	Observed deviation (G.).	ε.	Apparent zenith-distance.	Apparent semi-diameter.	log (g).	log (ε).	log (σ).	log (G corr.).	Sidereal time.	ε.	Apparent semi-diameter.
	h m		° ′	°	′ ″					h m	° ′	′ ″
Aug. 15.	22 3	107·6	+54 20	64·9	14 59·2	9·9998	9·9977	0·0008	2·0302	22 45·0	+54 35	15 0·2
* *gl*†	22 30	7·1	+54 26	62·9	14 59·6	0·0000	9·9987	0·0004	0·8504			
	22 45	109·1	+54 35	60·2	15 0·2	9·9998	0·0000	0·0000	2·0376			
gl	23 9	10·5	+54 43	57·8	15 0·7	0·0000	0·0013	9·9996	1·0221			
	23 29	112·1	+54 50	56·2	15 1·0	9·9998	0·0024	9·9992	2·0510			
gl	23 58	11·9	+55 0	54·0	15 1·5	0·0000	0·0039	9·9986	1·0780			
Aug. 16.	22 51	88·8	+65 34	60·6	14 56·1	9·9999	9·9982	0·0010	1·9475	23 26·0	+65 47	14 57·1
gl	23 6	9·5	+65 40	58·7	14 56·5	0·0000	9·9990	0·0006	0·9773			
	23 26	95·2	+65 47	56·4	14 57·1	9·9999	0·0000	0·0000	1·9785			
gl	23 46	9·7	+65 54	54·3	14 57·4	0·0000	0·0009	9·9996	0·9873			
	0 1	98·7	+65 59	52·9	14 57·8	9·9999	0·0016	9·9992	1·9950			
gl	0 16	9·7	+66 4	51·6	14 58·0	0·0000	0·0022	9·9990	0·9880			
Aug. 19.	0 9	34·8	+99 0	60·0	15 1·4	0·0000	9·9954	0·0012	1·5383	0 50·0	+99 16	15 2·7
‡ *gl*	0 26	3·1	+99 7	57·5	15 2·0	0·0000	9·9978	0·0006	0·4863			
	0 46	55·3	+99 15	54·7	15 2·6	0·0000	9·9997	0·0000	1·5311			
gl	1 11	2·6	+99 24	51·4	15 3·2	0·0000	0·0024	9·9904	0·4139			
	1 33	55·6	+99 32	48·5	15 3·8	0·0000	0·0047	9·9088	1·5375			
Oct. 4.	20 53	96·7	−54 22	72·5	15 27·0	9·9998	0·0023	0·0001	1·9876	21 38·0	−54 5	15 27·1
§ *gl*	21 23	13·8	−54 11	72·1	15 27·1	0·0000	0·0008	0·0000	1·1407			
	21 43	98·8	−54 3	72·2	15 27·1	9·9998	9·9998	0·0000	1·9944			
gl	22 5	12·9	−53 55	72·4	15 27·0	0·0000	9·9988	0·0001	1·1095			
	22 17	105·4	−53 50	72·7	15 26·9	9·9998	9·9981	0·0002	2·0210			
Oct. 9.	22 33	349·3	+ 6 31	61·3	14 56·9	9·9979	9·9993	0·0012	2·5416	23 33·0	+ 6 40	14 58·1
‖ *gl*	23 3	62·7	+ 6 36	58·2	14 57·6	9·9999	9·9997	0·0004	1·7973			
	23 25	370·4	+ 6 39	56·1	14 58·0	9·9976	0·0000	0·0000	2·5663			
gl	23 43	61·4	+ 6 42	54·6	14 58·3	9·9999	0·0002	9·9998	1·7881			
	23 56	389·6	+ 6 44	53·7	14 58·5	9·9973	0·0004	9·9996	2·5879			
gl	0 8	62·0	+ 6 46	52·8	14 58·7	9·9999	0·0006	9·9994	1·7923			
	0 20	385·9	+ 6 48	52·2	14 58·8	9·9974	0·0007	9·9992	2·5838			
Oct. 10.	22 45	** 337·8	+15 56	61·5	14 52·6	9·9980	9·9988	0·0008	2·5252	23 53·0	+16 8	14 53·5
¶ *gl*	23 5	47·4	+16 3	59·0	14 53·1	0·0000	9·9995	0·0004	1·6757			
	23 25	311·3	+16 9	56·7	14 53·6	9·9983	0·0001	9·9998	2·4914			
gl	23 47	46·2	+16 16	54·3	14 54·1	0·0000	0·0009	9·9996	1·6651			
	0 0	321·4	+16 20	52·8	14 54·4	9·9982	0·0013	9·9992	2·5057			

* Aug. 15.—N.E. wind all night; hazy sky; the moon was orange-coloured when low. No mention of the height of the directive magnet; it is assumed to have been at 3·00 in., as was the case on the following night.

† $22^h\ 30^m$. Galvanometer very unsteady.

‡ Aug. 19.—The scale was $38\frac{1}{2}$ inches from the galvanometer; therefore the readings have been multiplied by $\frac{36 \cdot 0}{38 \cdot 5}$ to reduce them to the usual distance of the scale, and by 0·6579 to reduce them to a height of 3·00 in. of the directive magnet.

§ Oct. 4.—New speculum used for the first time. Magnet at 3·46 in., observations reduced to 3·00 in. Apparently clear, but still a slight fog which makes the moon's image faint although still sharp.

‖ Oct. 9. Magnet at 3·46 in., observations reduced to 3·00 in.

¶ Oct. 10.—Magnet at 3·46 in., observations reduced to 3·00 in.

** The single readings vary more than 20 per cent.

Observations without glass plate; reductions to zenith.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1870.	Apparent zenith-distance.	Observed G. corr.	G. calc.	C—O.	*p.e.*	Zenith-mean (*z*).	*p.e*$_z$.
Aug. 8.	76°·4	98·2	87·2	−11·0	± 6·6	154·4	± 6·8
*	76·5	83·1	86·8	+ 3·7			
	77·0	77·2	84·8	+ 7·6			
Aug. 10.	73·3	162·5	......	......	......	256·0	......
*							
Aug. 11.	70·0	172·9	164·1	− 8·8	± 9·0	234·8	± 8·4
*	69·6	166·3	165·8	− 0·5			
	69·7	156·1	165·4	+ 9·3			
Aug. 12.	75·8	132·8	135·7	+ 2·9	± 1·8	234·3	± 1·2
*	67·5	172·1	173·7	+ 1·6			
	66·2	181·1	178·3	− 2·8			
	65·5	181·9	180·7	− 1·2			
Aug. 13.	78·2	74·4	89·2	+14·8	± 6·2	172·8	± 3·9
*	73·6	102·4	108·6	+ 6·2			
	69·7	124·5	121·7	− 2·8			
	62·0	145·7	141·1	− 4·6			
	61·1	149·7	142·8	− 6·9			
Aug. 14.	70·3	146·2	133·6	−12·6	± 8·7	192·7	± 6·7
*	66·9	147·0	144·6	− 2·4			
	62·9	142·1	155·2	+13·1			
Aug. 15.	64·9	107·2	103·1	− 4·1	± 2·4	132·3	± 1·6
*	60·2	109·0	110·5	+ 1·5			
	56·2	112·5	114·8	+ 2·3			
Aug. 16.	60·6	88·6	90·9	+ 2·3	± 1·4	109·2	± 0·9
	56·4	95·2	94·7	− 0·5			
	52·9	98·9	97·2	− 1·7			
Aug. 19.	60·0	34·5	32·7	− 1·8	± 1·1	39·1	± 0·7
*	54·7	34·0	34·3	+ 0·3			
	48·5	34·5	35·8	+ 1·3			
Oct. 4.	72·5	97·2	100·2	+ 3·0	± 3·2	153·8	± 2·8
*	72·2	98·7	101·1	+ 2·4			
	72·7	105·0	99·6	− 5·4			
Oct. 9.	61·3	348·0	353·2	+ 5·2	± 4·0	428·5	± 2·3
*	56·1	368·4	372·1	+ 3·7			
	53·7	387·2	379·1	− 8·1			
	52·2	383·5	383·2	− 0·3			
Oct. 10.	61·5	*335·2	307·5	−27·7	±15·9	374·1	±10·6
*	56·7	310·0	323·2	+13·2			
	52·8	320·4	333·2	+12·8			

* The remarks will be found at pp. 609 and 610.

Observations with glass plate; reductions to zenith.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.
1870.	Apparent zenith-distance.	Observed G. corr.	G. calc.	C−O.	*p.e.*	*z'*.	*p.e_z*.
Aug. 8.	77°·4	11·0	......	......	......	20·4	......
Aug. 10.	73·3	36·0	......	......	......	56·7	......
Aug. 11.	69·8 69·5	34·7 32·3	33·4 33·6	−1·3 +1·3	±1·3	47·5	±1·3
Aug. 12.	66·8 65·8	27·9 32·0	29·7 30·3	+1·8 −1·7	±1·7	39·5	±1·6
Aug. 13.	75·0 71·6 61·5	15·0 16·5 21·1	15·1 16·9 20·7	+0·1 +0·4 −0·4	±0·3	25·2	±0·2
Aug. 14.	68·6 65·4	14·5 15·4	14·4 15·4	−0·1 0·0	±0·1	20·0	±0·1
Aug. 15.	62·9 57·8 54·0	7·1 10·5 12·0	9·4 10·0 10·3	+2·3 −0·5 −1·7	±1·4	11·7	±0·9
Aug. 16.	58·7 54·3 51·6	9·5 9·7 9·7	9·3 9·7 9·9	−0·2 0·0 +0·2	±0·1	11·0	±0·1
Aug. 19.	57·5 51·4	3·1 2·6	2·8 2·9	−0·3 +0·3	±0·3	3·2	±0·2
Oct. 4.	72·1 72·4	13·8 12·9	13·4 13·3	−0·4 +0·4	±0·4	20·3	±0·4
Oct. 9.	58·2 54·6 52·8	62·7 61·4 62·0	60·5 62·3 63·2	−2·2 +0·9 +1·2	±1·3	70·9	±0·8
Oct. 10.	59·0 54·3	47·4 46·2	45·8 47·7	−1·6 +1·5	±1·5	54·2	±1·2

Before these results can be compared with each other, it is necessary to examine into another source of uncertainty, which, however, as will be seen, can not only be got rid of, but even made to yield a most interesting result. During the month of August an old and consequently tarnished speculum was used; this was replaced by one of great brilliancy before the observations of October were commenced. Hence the readings taken in August are below, and those in October above what they would have been with a mirror of ordinary working quality. Taking these facts into consideration, the observations were divided into two groups, according as they were made with the old or new speculum; and by comparing these groups with the Phase Table for 1871–72, at p. 605, the following values of speculum factors for reducing the observations to the standard mirror of those years were obtained. The weight which has been attached to the factor of each day varies directly as the square root* of the number of sets, and inversely as the reduction to the zenith, October 4th having only half weight on account of bad weather.

Observations without glass plate.

1870.	Speculum-factor.	Weight.
Aug. 8 . . .	1·806	0·9677
10 . . .	1·560	0·8982
11 . . .	1·530	1·2170
12 . . .	1·298	1·4270
13 . . .	1·456	1·5620
14 . . .	1·046	1·3010
15 . . .	1·185	1·4330
16 . . .	1·140	1·4940
19 . . .	1·314	1·5190

Whence August speculum-factor without glass plate=1·345; weight 11·82.

In the same way:—

1870.	Speculum-factor.	Weight.
Oct. 4	1·090	0·5645
„ 9	0·822	1·7360
„ 10	0·833	1·4880

Whence October speculum-factor without glass plate=0·866; weight 3·79 †.

These factors may also be deduced, though with less certainty, from the experiments with the glass plate for those elongations within the range of the observations of 1871–72.

1870.	Speculum-factor.	Weight.
Aug. 8	1·818	0·5386
„ 10	1·300	0·6351
„ 11	1·030	0·9970
„ 12	0·880	1·0740

Whence August speculum-factor with glass plate =1·164; weight 3·24.

In the same way:—

1870.	Speculum-factor.	Weight.
Oct. 4	1·042	0·4837
„ 9	0·722	1·1801
„ 10	0·661	1·0868

Whence October speculum-factor with glass plate =0·867; weight 2·75.

These four factors must be kept separate, because in 1870 the glass plate was always

* This ratio was used as tending to lessen the preponderance of any given day.

† Combining the factors for experiments without glass plate, it appears that a new speculum reflects 1·55, say *once and a half* as much heat as the most tarnished one that would be considered serviceable.

warmed at the fire in order to dissipate any accidental film of moisture, while in the subsequent years it was merely kept in a dry room and then briskly rubbed with cotton-wool before being placed in the telescope. The quantity of heat transmitted by the glass is greater in the former case than in the latter.

The results for 1870 can now be arranged like those for 1871–72; but, in addition to the reductions to a mean distance of the sun and moon, the speculum-factors must be applied. We have thus the following summary, where

$$\log \Sigma \text{ corr.} = \log \varrho^2 + \log R'^2 + \log \text{ speculum-factor}.$$

Observations reduced to a mean distance of the Sun and Moon compared with Curve.

1870.	ε.	log Σ corr.	z' (obs.).	No. of sets.	1871–72. z' (curve).	C−O.	$\frac{100}{z'}$(C−O).
Oct. 4.	−54° 5′	9·9443	135·3	3	170·3	+35·0	+25·9
Aug. 8.	−30 19	0·1217	204·4	3	274·3	+69·9	+34·2
„ 10.	− 5 59	0·1364	350·3	2	406·2	+55·9	+16·0
Oct. 9.	+ 6 40	9·9715	401·4	4	381·0	−20·4	− 5·1
Aug. 11.	+ 8 39	0·1448	327·7	3	372·8	+45·1	+13·8
Oct. 10.	+16 8	9·9756	353·7	3	339·9	−13·8	− 3·9
Aug. 12.	+20 19	0·1527	333·0	4	321·4	−11·6	− 3·5
„ 13.	+31 38	0·1610	250·3	5	271·1	+20·8	+ 8·3
„ 14.	+43 2	0·1681	283·8	3	220·6	−63·2	−22·3
„ 15.	+54 35	0·1722	196·6	3	173·2	−23·4	−11·9
„ 16.	+65 47	0·1737	163·0	3	138·4	−24·6	−15·1
„ 19.	+99 16	0·1674	57·4	3	56·0	− 1·4	− 2·4

z' (obs.) is the zenith-value of the total heat for a mean distance of the sun and moon and for an average speculum;

z' (curve) is taken from the curve derived from the results of 1871–72;

C—O is the difference between curve and observation, and $\frac{100}{z'}$(C—O) is the percentage difference.

The deviation of the total heat-results from the curve of 1871–72 are within such moderate limits, that, taking into consideration the way in which the speculum-factors were derived from it, it does not appear advisable to alter its course in order to attempt a still closer representation of the whole of the observations.

The following Table gives a summary of all the observations made with the glass plate interposed, during the years 1870, 1871, and 1872, of which the preliminary reductions have already been given.

Glass-plate observations reduced to a mean distance of the Sun and Moon, and compared with the Phase-Curves. (See also Plate XLVIII., Curve B.)

1800+	ε.	log z.	log Σ corr.	w=No. of sets.	z'' observation.	z'' curve.	C−O.	Percentage of moon's heat transmitted by glass.		Numbers of sets.
								Observations.	Curves.	
May 28, 71.	−72° 14′	1·0175	0·9926	3	10·2	10·2	0·0	10·1	9·2	6 and 3
Dec. 20, 71.	−66 8	1·1472	0·0016	1	14·1	13·4	− 0·7	10·0	10·4	7 ,, 1
Apr. 18, 72.	−56 11	1·2301	0·0221	2	17·9	18·7	+ 0·8	9·5	11·5	9 ,, 2
Oct. 4, 70.	−54 5	1·3084	9·9447	2	17·9	19·9	+ 2·0	13·2	11·7	3 ,, 2
Apr. 30, 71.	−53 46	1·4297	9·9835	5	25·9	20·0	− 5·9	11·8	11·7	6 ,, 5
May 30, 71.	−45 37	1·5109	9·9665	1	30·0	24·3	− 5·7	16·9	11·9	2 ,, 1
Mar. 21, 72.	−36 42	1·5169	0·0166	2	34·2	29·1	− 5·1	10·9	12·0	3 ,, 2
Aug. 8, 70.	−30 19	1·3086	0·0589	1	23·3	33·3	+10·0	11·4	12·1	3 ,, 1
Mar. 22, 72.	−24 27	1·5434	0·0098	3	35·8	39·6	+ 3·8	11·5	12·9	6 ,, 3
Apr. 3, 71.	−21 31	1·6879	9·9653	6	45·0	43·9	− 1·1	14·1	13·5	8 ,, 6
June 1, 71.	−17 29	1·7999	9·9491	3	56·1	50·6	− 5·5	15·8	14·4	4 ,, 3
May 3, 71.	−14 16	1·7340	9·9494	2	48·2	56·6	+ 8·4	13·7	15·2	2 ,, 2
Dec. 25, 71.	−10 16	1·7446	0·0242	4	58·7	63·8	+ 5·1	14·5	16·2	7 ,, 4
Aug. 10, 70.	− 5 59	1·7535	0·0600	1	65·1	69·4	+ 4·3	18·6	17·1	2 ,, 1
Mar. 24, 72.	− 4 9	1·7473	9·9945	1	55·2	70·2	+15·0	13·6	17·2	5 ,, 1
June 2, 71.	− 3 42	2·0083	9·9460	2	90·0	70·3	−19·7	20·1	17·3	3 ,, 2
Oct. 9, 70.	+ 6 40	1·8509	9·9714	3	66·4	61·4	− 5·0	16·6	16·1	4 ,, 3
Feb. 24, 72.	+ 8 33	1·6462	0·0043	2	44·7	58·7	+14·0	13·1	15·7	8 ,, 2
Aug. 11, 70.	+ 8 39	1·6768	0·0686	2	55·6	58·6	+ 3·0	17·0	15·7	3 ,, 2
Oct. 10, 70.	+16 8	1·7339	9·9759	2	51·3	48·9	− 2·4	14·5	14·4	3 ,, 2
Aug. 12, 70.	+20 19	1·5963	0·0768	2	47·1	44·2	− 2·9	14·2	13·8	4 ,, 2
Dec. 28, 71.	+23 28	1·5661	0·0211	4	38·7	40·6	+ 1·9	12·1	13·2	7 ,, 4
Aug. 13, 70.	+31 38	1·4020	0·0853	3	30·7	31·9	+ 1·2	12·3	11·8	5 ,, 3
,, 14, ,,	+43 2	1·3004	0·0929	2	24·7	22·2	− 2·5	8·7	10·1	3 ,, 2
,, 15, ,,	+54 35	1·0681	0·0975	3	14·6	16·2	+ 1·6	7·4	9·4	3 ,, 3
,, 16, ,,	+65 47	1·0417	0·0996	3	13·9	13·0	− 0·9	8·5	9·4	3 ,, 3
,, 19, ,,	+99 16	0·5067	0·0951	2	4·0	4·0	0·0	7·0	7·1	3 ,, 2

Phase Table for observations with glass plate. (See Plate XLVIII., Curve B.)

ε.	z''.		ε.	z''.		ε.	z''.		ε.	z''.	
−74°	9·2	1·1	−30°	33·5	1·9	+14°	51·5	2·4	+58°	15·1	0·6
72	10·3	1·1	28	35·4	2·2	16	49·1	2·3	60	14·5	0·5
70	11·4	1·0	26	37·6	2·6	18	46·8	2·3	62	14·0	0·6
68	12·4	1·1	24	40·2	2·9	20	44·5	2·2	64	13·4	0·5
66	13·5	1·1	22	43·1	3·2	22	42·3	2·3	66	12·9	0·5
64	14·6	1·0	20	46·3	3·4	24	40·0	2·2	68	12·4	0·6
62	15·6	1·1	18	49·7	3·6	26	37·8	2·2	70	11·8	0·5
60	16·7	1·1	16	53·3	3·8	28	35·6	2·1	72	11·3	0·5
58	17·8	1·0	14	57·1	3·7	30	33·5	2·0	74	10·8	0·6
56	18·8	1·1	12	60·8	3·5	32	31·5	2·0	76	10·2	0·5
54	19·9	1·1	10	64·3	3·0	34	29·5	1·8	78	9·7	0·5
52	21·0	1·0	8	67·3	2·1	36	27·7	1·7	80	9·2	0·6
50	22·0	1·1	6	69·4	+0·9	38	26·0	1·6	82	8·6	0·5
48	23·1	1·0	4	70·3	0·0	40	24·4	1·5	84	8·1	0·6
46	24·1	1·1	− 2	70·3	−0·8	42	22·9	1·4	86	7·5	0·5
44	25·2	1·1	0	69·5	1·8	44	21·5	1·3	88	7·0	0·5
42	26·3	1·0	+ 2	67·7	2·5	46	20·2	1·1	90	6·5	0·6
40	27·3	1·1	4	65·2	2·8	48	19·1	1·0	92	5·9	0·5
38	28·4	1·1	6	62·4	2·9	50	18·1	0·9	94	5·4	0·5
36	29·5	1·2	8	59·5	2·8	52	17·2	0·8	96	4·9	0·6
34	30·7	1·3	10	56·7	2·7	54	16·4	0·7	98	4·3	0·5
32	32·0	1·5	12	54·0	2·5	56	15·7	0·6	+100	3·8	
−30	33·5		+14	51·5		+58	15·1				

Comparison of the reducing curve in zenith-distance with LAPLACE'S *law.*

As the observations of the moon's radiant heat were all made within a range of little more than 50° of zenith-distance, it is of interest to see if any of the more generally received formulæ for the extinction of heat or light in our atmosphere agree with the Table at p. 598; for only in the event of this being the case can we extend our considerations to some interesting cases not included within that range.

The best form for such a formula is doubtless that first given by LAPLACE*, and which is to be found in POUILLET'S 'Météorologie' (t. ii. p. 711) in the modified form

$$t=ap^{\varepsilon}, \quad \ldots \quad (1)$$

where t is the heat-effect in a given interval of time, and ε the amount of air through which the rays of heat have passed; a and p are constants to be determined from the observations; ε is calculated on the assumption that the atmosphere forms a coat of uniform density of a thickness h, which is taken equal to unity. If any value, r, for the radius of the earth be assumed, then ε can be calculated for any zenith-distance, z, by the formula

$$\varepsilon=\sqrt{2rh+h^2+r^2\cos^2 z}-r\cos z.$$

Substituting logarithms in (1), we have

$$\log t=\log a+\varepsilon\log p,$$

where for $\log t$ we can take φz with its sign changed, and thus get a series of equations:—

$$\left.\begin{array}{l} -\varphi z \quad =\log a+\varepsilon\ \log p, \\ -(\varphi z)'=\log a+\varepsilon'\ \log p, \\ -(\varphi z)''=\log a+\varepsilon''\log p, \\ \&\text{c.} \qquad\qquad \&\text{c.} \end{array}\right\} \quad \ldots \quad (2)$$

whence $\log a$ and $\log p$ can be deduced. In his researches on the sun's radiant heat POUILLET assumed $r=80\,h$. In accordance with this the values of ε for every degree from 29° to 80° were calculated and the constants a and p deduced. It was then found that the values of φz were tolerably well represented, yet at the same time it was evident that $r=70\,h$ or $r=60\,h$ would fulfil the required conditions more accurately; and in point of fact $r=60\,h$ is very nearly the value that makes the outstanding errors in the representation of φz a minimum; $r=58\,h$ lessens the sum of the squares of the errors to an extent altogether immaterial. The following are the 51 equations of the form (2), where the coefficient of $\log p$ is

$$\varepsilon=\sqrt{121+3600\cos^2 z}-60\cos z.$$

* Mécanique Céleste, t. iv. livre x. chap. 3.

	$t-c.$
$-0{\cdot}0046 = \log a + 1{\cdot}141 \log p$	$-0{\cdot}0002$
$-0{\cdot}0056 = \log a + 1{\cdot}152 \log p$	1
$-0{\cdot}0067 = \log a + 1{\cdot}163 \log p$	2
$-0{\cdot}0078 = \log a + 1{\cdot}176 \log p$	1
$-0{\cdot}0090 = \log a + 1{\cdot}188 \log p$	2
$-0{\cdot}0102 = \log a + 1{\cdot}202 \log p$	1
$-0{\cdot}0115 = \log a + 1{\cdot}216 \log p$	1
$-0{\cdot}0129 = \log a + 1{\cdot}231 \log p$	1
$-0{\cdot}0143 = \log a + 1{\cdot}246 \log p$	2
$-0{\cdot}0158 = \log a + 1{\cdot}263 \log p$	1
$-0{\cdot}0174 = \log a + 1{\cdot}280 \log p$	1
$-0{\cdot}0191 = \log a + 1{\cdot}298 \log p$	2
$-0{\cdot}0209 = \log a + 1{\cdot}317 \log p$	2
$-0{\cdot}0227 = \log a + 1{\cdot}337 \log p$	1
$-0{\cdot}0247 = \log a + 1{\cdot}358 \log p$	2
$-0{\cdot}0268 = \log a + 1{\cdot}380 \log p$	3
$-0{\cdot}0290 = \log a + 1{\cdot}403 \log p$	4
$-0{\cdot}0314 = \log a + 1{\cdot}427 \log p$	5
$-0{\cdot}0339 = \log a + 1{\cdot}453 \log p$	6
$-0{\cdot}0365 = \log a + 1{\cdot}480 \log p$	7
$-0{\cdot}0392 = \log a + 1{\cdot}508 \log p$	9
$-0{\cdot}0420 = \log a + 1{\cdot}538 \log p$	9
$-0{\cdot}0449 = \log a + 1{\cdot}570 \log p$	8
$-0{\cdot}0479 = \log a + 1{\cdot}603 \log p$	8
$-0{\cdot}0510 = \log a + 1{\cdot}638 \log p$	6
$-0{\cdot}0542 = \log a + 1{\cdot}676 \log p$	$-0{\cdot}0003$
$-0{\cdot}0575 = \log a + 1{\cdot}715 \log p$	$0{\cdot}0000$
$-0{\cdot}0609 = \log a + 1{\cdot}757 \log p$	$+0{\cdot}0004$
$-0{\cdot}0645 = \log a + 1{\cdot}802 \log p$	10
$-0{\cdot}0683 = \log a + 1{\cdot}849 \log p$	15
$-0{\cdot}0725 = \log a + 1{\cdot}900 \log p$	20
$-0{\cdot}0771 = \log a + 1{\cdot}953 \log p$	23
$-0{\cdot}0823 = \log a + 2{\cdot}010 \log p$	24
$-0{\cdot}0881 = \log a + 2{\cdot}072 \log p$	$+0{\cdot}0023$

	$t-c$.
$-0.0945 = \log a + 2.137 \log p$	$+0.00019$
$-0.1014 = \log a + 2.207 \log p$	15
$-0.1088 = \log a + 2.283 \log p$	11
$-0.1167 = \log a + 2.364 \log p$	7
$-0.1254 = \log a + 2.452 \log p$	$+0.0002$
$-0.1348 = \log a + 2.547 \log p$	-0.0005
$-0.1449 = \log a + 2.650 \log p$	11
$-0.1557 = \log a + 2.762 \log p$	15
$-0.1673 = \log a + 2.884 \log p$	18
$-0.1797 = \log a + 3.017 \log p$	20
$-0.1930 = \log a + 3.163 \log p$	18
$-0.2074 = \log a + 3.324 \log p$	13
$-0.2232 = \log a + 3.501 \log p$	8
$-0.2407 = \log a + 3.697 \log p$	-0.0001
$-0.2603 = \log a + 3.915 \log p$	$+0.0004$
$-0.2823 = \log a + 4.157 \log p$	8
$-0.3069 = \log a + 4.428 \log p$	$+0.0012$;

whence

$$51.00 \log a + 101.7900 \log p - 4.2542 = 0,$$
$$101.79 \log a + 240.8824 \log p - 11.9755 = 0;$$

whence

$$\log a = 0.10096 ; \log p = 9.90762 - 10.$$

If these values of log a and log p are substituted in the above fifty-one equations, the equations will be found to be satisfied, excepting the $t-c$ (table — calculation) given on the right of the page.

The smallness of these differences naturally leads us to regard the formula as trustworthy for those circumstances under which it has hitherto been impossible to procure observations, and even for those cases where they are altogether beyond our reach.

For taking the formula (1),

$$t=ap^{\varepsilon},$$

we have only to put $\varepsilon=1$ in order to get

$$t=ap,$$

or the zenith-effect of the moon's heat; and further, by taking the extreme case $\varepsilon=0$, we have

$$t=a,$$

or the heat-effect supposing the atmosphere to be removed altogether*.

* Taking the moon's maximum zenith heat-effect at the earth's surface at 407·3 (p. 606), we have 513·9 as the maximum before her rays enter our atmosphere.

It will be remembered that the value $\phi z=0\cdot0046$ for $z=29^{\circ}$ was obtained by comparison with SEIDEL'S results, based on the comparison of observations of the light of stars near the zenith with that of others at various altitudes.

Formula (1) now enables us to find this quantity independently of SEIDEL'S results; for since ϕz is the logarithmic factor for reducing to the zenith it must become zero for $\varepsilon=1$, or we have the new condition

$$\log a+\log p=0\,;$$

it is, however, 0·00858, which must be added to all the values of ϕz and subtracted from $\log a$ in order to fulfil the above condition.

Simultaneous Observations of the Moon's Heat and Light during the Eclipse of November 14th, 1872.

During the partial eclipse of the moon of November 14th, 1872, an attempt was made to ascertain whether or not the lunar surface required an appreciable time to acquire the temperature due to the action of the sun's light shining on it at the moment.

Obviously a total eclipse would be a favourable time for determining this point (at least for heat which has penetrated to a small depth only), as the transition from light to darkness is so much more rapid than that caused by the moon's rotation on her axis; but owing to the smallness of the eclipse (there being little more obscuration than that due to the penumbra alone), the considerable decrease of the moon's altitude towards the close of the eclipse, and still more to the uncertainty of the weather (which allowed us only a few glimpses now and again of the moon), the results only go so far as to show that the heat was diminished during the eclipse in a rather greater proportion than the light. The minimum for both heat and light occurred at or very near the middle of the eclipse.

The following Tables, in which the columns correspond respectively to those with the same heading in the Tables already given, contain full particulars of the observations and their reductions.

The moon's light was measured with a ZÖLLNER'S photometer, with which, as is well known, the total light of the observed body is, *cæteris paribus*, proportional to the square of the sine of the reading of the intensity-circle. This reading is given in column I. (see next page). This being borne in mind, the meaning of the other columns will be readily understood.

Observations of the Moon's Heat, Nov. 14th, 1872.

Sidereal time.	Mean deviation (G).	The Moon's ε.	The Moon's Zenith-distance.	The Moon's Apparent semidiameter.	log (g).	log (σ).	ϕz.	Zenith-mean (z).
h m		° ′	°	′ ″				
6 15	479·0	−2 7	50·2	15 40·0	9·9960	0·9970	0·0426	520·0
7 47	264·7	−1 40	62·4	15 37·5	9·9977	0·9994	0·0906	323·9
8 32	186·4	±1 34	68·9	15 35·7	9·9994	0·0010	0·1439	259·8
8 45	182·5	±1 34	70·8	15 35·2	9·9994	0·0014	0·1649	267·3
9 6	198·6	+1 32	73·9	15 34·4	9·9993	0·0022	0·2059	320·2

The moon's tabular semidiameter . . . =15′ 36″·6

The moon's adopted apparent semidiameter =15 30·0

Observations of the Moon's light, November 14th, 1872.

Sidereal time.	I.	Log sin² I.	Log. corr. (σ).	Zen.-dist.	Moon's app. semidiameter.	φz.	Moon's light.
h m	°			°	′ ″		
6 14½	49·5	9·762	9·997	50·1	15 40·0	·045	0·6368
6 18½	47·5	9·735	9·997	50·6	15 40·0	·047	0·6012
7 46½	34·65	9·510	0·000	62·4	15 37·3	·116	0·4227
7 50	32·15	9·452	0·000	62·9	15 37·2	·120	0·3733
8 28½	31·0	9·424	0·001	68·5	15 35·8	·175	0·3981
8 33½	29·05	9·372	0·001	69·2	15 35·6	·182	0·3589
8 37½	29·2	9·377	0·001	69·8	15 35·5	·189	0·3690
*8 41½	23·95	9·217	0·001	70·3	15 35·3	·195	[0·2588]
8 45½	29·5	9·385	0·001	70·9	15 35·2	·203	0·3882
8 49½	31·4	9·434	0·001	71·4	15 35·0	·210	0·4416
9 8½	29·35	9·380	0·002	74·2	15 34·2	·253	0·4295
9 12½	27·6	9·332	0·002	74·8	15 34·2	·264	0·3963

Interpolating the light-observations and multiplying them by the common factor 764·9 to reduce them to the same scale as the heat-observations, the following comparison of the moon's heat and light is arrived at.

Comparison of the Moon's Heat and Light.

Sidereal time from middle of eclipse.	Moon's heat.	Moon's light.	$l-h$.
h m			
−2 11	520·0	477·6	−42·4
−0 39	323·9	309·0	−14·9
+0 6	259·8	295·2	+35·4
+0 19	267·3	299·3	+32·0
+0 40	320·2	309·8	−10·4

It may be well to add that although the thermopiles used on this day were the same as for the rest of the experiments, they had been remounted in the early summer, by which their sensitiveness was considerably increased. The allowance for the extinction of the moon's heat in the atmosphere was made in accordance with the Table deduced in the earlier part of this paper, while that for the moon's light was taken from Professor SEIDEL'S Table.

Comparison of the Phase-Curve for Heat as given in this paper with that for Light deduced by Professor ZÖLLNER *from his and Sir* JOHN HERSCHEL'S *observations.*

A diagram (Plate XLVIII., Curve C) accompanies this paper, on which is laid down the heat-curve and the determinations for the moon's light, those marked Z and Z2 derived from observations made by Professor ZÖLLNER by his first and second methods taken from the Table at p. 102 of the 'Photometrische Untersuchungen,' and those marked H derived from Sir JOHN HERSCHEL'S observations taken from plate iv. of that work.

* Some mistake in reading the intensity-circle; observation rejected.

Four determinations made by Professor ZÖLLNER after his diagram was engraved are here given, and two errors pointed out by him* in the elongations of two of Sir JOHN HERSCHEL'S observations (caused by errors in the 'Cape Observations') have been rectified. The dotted line is Professor ZÖLLNER'S calculated curve, the ordinates of which have been increased in the ratio of 4·880 to 1, so as to make it agree as closely as possible with the heat-curve†.

On inspecting the diagram, it is at once apparent that the increase of the moon's light in approaching the full moon is more rapid than that of her heat, so much so that ZÖLLNER resorted to a cusped curve for its representation. The introduction of the additional observations, however, three of which are upper ones in elongations −27°, −24°, and −11°, would not only seem to indicate a necessity for rounding off the maximum of the light-curve, but also for placing it slightly before full moon, and thus making it agree with the heat-curve in this remarkable feature.

Some uncertainty appears to arise on account of the employment by ZÖLLNER of a photometer in which, when used on the moon, the light has to traverse a system of Nicol prisms; but it does not appear clear in what azimuth the system was turned during each observation, and consequently it is not known whether the correction due to this cause would diminish the departure of the heat- from the light-curve, or the reverse‡.

Attempt to compare the Moon's Radiant Heat with that from a terrestrial source.

The effect of the moon's heat has hitherto been expressed in this paper on a purely arbitrary scale, namely by the differences of the readings of one and the same galva-

* Photometrische Untersuchungen, p. 175, note.

† In accordance with the heading of the Table (Phot. Unt. p. 198) the curve is kept within the limits of 50° before and 70° after full moon.

‡ Taking the extreme case, where the plane in which the sun, moon, and earth, and therefore the plane in which the moon's light is polarized, is parallel to the plane of reflection of the transparent plate of parallel glass in the photometer, the system of Nicols being supposed to be set with the principal axis parallel to this plane, the phase-curve obtained would differ from the heat-curve by only about two thirds the present amount. Were the system of Nicols moved round through 90°, the correction to be applied to the curve would be in the other direction.

It has been assumed that the ratio of the two components of natural light after passing through the plate of glass is 0·84 to 1·00 at 45° incidence, and the corresponding ratio for moonlight 0·83 to 1·00 (probably too high an estimate of the mean polarization of the moon's light), at quadrature the maximum polarization occurring at about 77° elongation. The ratio of intensities found in the two cases would thus be the same at about 85° from full moon, and their ratio 0·84 to 1·00 at full moon. Sir J. HERSCHEL employed a photometer which appears to be free from this source of error.

I cannot find that any one has devoted much attention to the subject of the polarization of light from the moon except ARAGO and Father SECCHI. The former states that the maximum polarization occurs at or near quadrature, but gives no estimate of its amount. Our experiments in this direction are not as accordant as might be wished, and for the present do not appear worth publishing.

It may, however, be well that any who happen to be working at Photometry should have their attention in the mean time called to this possible source of error.

nometer under certain conditions of adjustment, when acted on by the electric currents generated by the moon's heat, which, falling on the large mirror of the 3-foot telescope, is concentrated by concave mirrors of short focal length alternately on each of a given pair of thermopiles. It was therefore desirable to compare the effect thus produced by the moon's heat with that of a given terrestrial source (say, a blackened tin vessel) acting under circumstances as nearly as possible similar to those in the case of the moon. The great focal length of the telescope (27 feet) altogether precluded the use of the whole instrument for this purpose, so the condensing-mirrors with the thermopiles were detached from the telescope, and each separate pile was exposed to the alternate action of two circular surfaces of blackened tin backed by water of different temperatures, these surfaces being alternately exposed for the interval of a minute through a circular aperture in a fixed wooden screen.

In this way, by varying the temperature of the water, a considerable range of readings of the galvanometer was obtained; and these being compared, by means of an empirical formula it was easy to calculate the reading for any given temperatures of the tins.

In this way, by taking into account the moon's apparent semidiameter, the effective area and reflective power of the 3-foot speculum, and the action of the *two* piles on the one hand, and the distance and radius of the circular aperture, the effective area of the condensing-mirror, and the *one* pile on the other, the temperature of the tin vessel necessary to produce an effect equal to that of the full moon, when acting under similar circumstances, could be at once calculated.

In the case of the moon-observations we have the following particulars:—

	in.
Full area of speculum	=973·12
Deduct for direct obstruction . . .	=116·80
	856·32

The assumed reflective power of the speculum	=0·5325
Proportion of the cone of rays not intercepted by pile and holder	=0·90064
Hence the 3-foot speculum may be replaced by one of perfect reflective power whose area	=410·7 sq. in.
Moon's mean apparent semidiameter	=15′·557

If we take 1′ as the unit,

$$\text{the moon's area} = \pi \times 15{\cdot}557^2,$$

and the source of lunar radiation for a unit of excess of temperature

$$=\pi\,.\,(15{\cdot}557)^2 \times 410{\cdot}7 = \text{M}.$$

Again, the distance of the hot tin from the condensing-mirrors was 111 inches, its radius 6 inches, and the unobstructed area of the condensing-mirror 8·597 inches, therefore source of heat for comparison $=\pi(0{\cdot}054054 \times 3437{\cdot}75)^2 \times 8{\cdot}597 = \text{T}$; hence

$$\frac{\text{T}}{2\text{M}} = 1{\cdot}4933.$$

That is, under the circumstances stated above, a difference of temperature of the blackened tins acting alternately on *one* pile produces 1·4933 times the effect that the same excess of temperature of the area occupied by the moon's disk above that of an equal area of the neighbouring sky (assumed of the same temperature as the colder tin) would do acting on the *two* piles in their position in the telescope.

It has been shown (p. 618, note) that the moon at full, or, more strictly speaking, at the time of its maximum heat, before its rays traverse our atmosphere, would produce an effect on the galvanometer of 513·9 parts of the scale; therefore the tin at the same excess of temperature (the radiating powers of the two surfaces being assumed equal) would give $513{\cdot}9 \times 1{\cdot}4933 = 767{\cdot}4$ parts.

Therefore, employing Dulong and Petit's formula for the velocity of cooling*,

$$V = ma^{\theta}(a^{t} - 1)$$

(where θ is the temperature in degrees Centigrade of the colder tin, $t+\theta$ that of the hotter one, a a constant$=1{\cdot}0077$, V the mean difference of consecutive pairs of readings of the galvanometer, and m a constant deduced from experiments with the tins$=558{\cdot}0$†), a mean value of $\theta=45°$ Fahr. (7°·22 Cent.) will give $V=767{\cdot}4$ when $t=197°{\cdot}5$ Fahr. (109°·7 Cent.)‡.

This result, it will be observed, differs much from a rough estimation of the value of the scale-readings given at the conclusion of a former communication§. Probably this may be caused principally by neglect in distinguishing between the effect of one pile and of both piles; in other words, omitting a factor 2 from the former calculation, the

* Annales de Chim. et de Phys. t. vii. This formula is, strictly speaking, only applicable to a radiating body *in vacuo*; but for the comparatively moderate temperatures here dealt with it is perhaps as correct as any other.

† These experiments were made on March 25th and 26th, 1872, and are as follows, where V is the mean of 10 consecutive differences:—

	θ (Cent.).	t.	V obs.	V calc.	C−O.	
Pile A.	5°67	24°44	110·4	113·9	+3·5	
	6·06	21·83	100·7	100·9	+0·2	
	7·11	40·11	198·8	200·9	+2·1	$m=534{\cdot}3$
	7·50	34·39	172·3	169·0	−3·3	
	7·89	29·78	145·7	144·1	−1·6	
	8·33	24·39	116·4	115·9	−0·5	
Pile B.	5·78	23·11	110·4	116·7	+6·3	
	6·17	20·50	95·6	102·8	+7·2	
	6·72	43·22	241·4	238·0	−3·4	
	7·33	36·95	202·7	199·4	−3·3	$m=581{\cdot}7$
	7·72	32·22	170·2	171·1	+0·9	
	8·00	28·17	148·1	147·5	−0·6	
	8·28	26·22	137·1	136·6	−0·5	

‡ If Newton's law ($V=Ct$) be employed, the corresponding excess of temperature of the tin would come ou 152°·2 Cent.=274° Fahr.

§ Proceedings of the Royal Society, No. 112 (1869), p. 443.

details of which have not been preserved. Although the result now given has been worked out with every care, it must still be considered simply an attempt to connect the readings of the galvanometer with a constant known source of heat; and we must admit that the problem of the determination of the lunar temperature is nearly as far removed as ever from our grasp. The formula we have made use of is admitted to be a purely empirical one, based on experiments made within very limited ranges of temperature, within which it appears to agree more closely with the observed law than NEWTON'S more simple formula. Nevertheless we can obviously feel no assurance that it is generally true, nor indeed can we employ it, like NEWTON'S, to determine the excess of the temperature of the hotter over that of the cooler body, without knowing one or other of the two temperatures.

I feel that I should not conclude this paper without bearing testimony to the energy and perseverance with which my assistant, Dr. RALPH COPELAND, has conducted the observations which form the subject of the greater part of it, as also to the great care and the ability with which he has worked out the reductions. I had hoped to have sent in this paper before the close of last Session; but the greater completeness which is the result of the delay, as well as the fact that the exceptionally unpropitious state of the weather would have prevented our profiting, in a new series of observations by any suggestions which might have come to us, have removed all cause for regretting this. One thing I should have much wished to have added to this investigation had there been any reasonable prospect of doing so without considerably more delay—namely, a more satisfactory determination of the value of the scale, by means of a comparison of the deviations due to solar with those due to lunar radiation, than that given in a former communication based on observations made with an uncertain sky and of only two or three hours' duration.

APPENDIX.

Received May 7, 1873.

Since the foregoing was written an explanation has suggested itself for the very considerable divergence (more than could fairly be ascribed to errors of observation) of our heat-curve from Professor ZÖLLNER'S light-curve for change of phase, which will at the same time account for the increase of the percentage of heat transmitted by glass towards the time of full moon.

Were it not for the negative result obtained with the thermopile on March 24th, 1871*, the latter fact might perhaps have been attributed to a constant amount of heat emitted from the interior of the moon; but now this explanation is no longer admissible.

On referring to the Table at page 615 of the foregoing paper, it will be seen that the percentage of heat transmitted by glass increases gradually from each side to a maximum at or near the time of full moon; consequently the phase-curve for "heat through glass"

* See page 589, note.

will rise more rapidly towards full moon than the total heat-curve, and should therefore diverge less from ZÖLLNER's than the total heat-curve, when the ordinates are increased by a suitable factor.

This comparison has accordingly been made. The ordinates of the curve B (Plate XLVIII.) have been multiplied by 5·7916, to make the curve correspond in average height with curve A (Plate XLVIII.); and, with the same object, ZÖLLNER's photometric determinations obtained with two different photometers and those by Sir JOHN HERSCHEL with the prism photometer were multiplied respectively by the following factors*:—

ZÖLLNER's 1st method	3·8471
,, 2nd (or improved) method	3·8304
HERSCHEL's	4·2920

Applying these factors to the numbers given at page 102 of the 'Photometrische Untersuchungen' (for ZÖLLNER's observations), and to the quantities taken from plate iv. of that work for HERSCHEL's results, the following numbers were obtained:—

Professor ZÖLLNER; 1st method.

ε.	Moon's light.	ε.	Moon's light.
-40°	181·2	$+13^\circ$	317·7
−28	217·1	+27	222·7
− 8	354·7	+42	146·7
− 1	379·3	+69	56·1
+ 5	335·5		

Professor ZÖLLNER; 2nd method.

ε.	Moon's light.	ε.	Moon's light.
-70°	77·8	-19°	262·0
−58	103·9	−11	339·9
−46	138·3	+28	218·3
−41	168·2	+39	159·7
−33	187·1	+52	111·5
−27	243·1	+62	78·1
−24	273·4		

* The necessity for applying different factors to render comparable groups of observations already similarly treated by ZÖLLNER, arises from the fact that ZÖLLNER, since he constructed his diagram, has added four observations by his second method, and has discovered two errors of 10° each in the elongation of the moon as given in the 'Cape Observations.'

Sir JOHN HERSCHEL.

ε.	Moon's light.	ε.	Moon's light.
−75°	81·6	−13°	269·6
−59	127·0	− 1	417·3
−51	112·9	+13	300·4
−48	144·6	+28	244·6
−46	134·4	+41	171·7
−39	151·1	+44	166·2
−37	165·3	+82	52·4
−19	243·0		

Then tracing with a perfectly unbiassed mind the curve given in Plate XLVIII. (Curve D), and not until afterwards superposing the curve of "heat through glass," treated as already stated, there was found to be a fair agreement between the two curves.

For the luminous rays, then, from the moon, the results obtained with the eye aided by the photometer and those derived from the indications of the thermopile are as nearly identical as could be expected; and it seems just to seek for the explanation of the far greater divergence (see Plate XLVIII., Curve C), under more favourable circumstances of observation, of the "total heat-curve" in a real difference between the laws which govern the emission of heat and light from the lunar surface.

Let us for the moment assume with ZÖLLNER that the moon's surface is covered with angular ridges, whose sides are planes of, say, 52° inclination, and whose direction is perpendicular to the plane in which the earth, sun, and moon lie. The sun's light will then in many parts shine on one side only of each of these ridges, which will reflect or diffuse the incident light diminished by the amount absorbed. Let $1-\mu=$ quantity absorbed, $\mu=$ that emitted.

Some of the latter will strike the shaded sides of the ridges, but of this $1-\mu$ will be absorbed and only μ emitted. With the heat, however, this will not be the case. If the moon's temperature be assumed from moment to moment practically constant, the *whole* of the heat which falls on her surface must necessarily leave it again; whereas for every unit of light and heat which falls on the surface, of the former only μ leaves it after one reflection, μ^2 after two reflections, μ^3 after three reflections, and so on. Therefore the proportion of heat emitted by the shadows will be, as compared with the light coming from those same parts, $\frac{1}{\mu}$ times greater than what comes from the parts in direct sunlight; the heat emitted in directions removed some distance from the sun will be larger compared with that thrown back more towards the sun, and the greater flatness of the heat-curve and the increase of percentage of heat transmitted by glass at or near full moon are at once explained.

The complete solution of the question would probably be complicated, and, owing to the very unequal distribution of mountain and plain, perhaps unprofitable, even if we possessed fuller data than we at present have on which to base our calculations.

A further, but probably a less important, cause for the divergence of the heat- and light-curves lies in the fact that the moon's surface to a certain extent "regularly reflects" as well as diffuses light and heat.

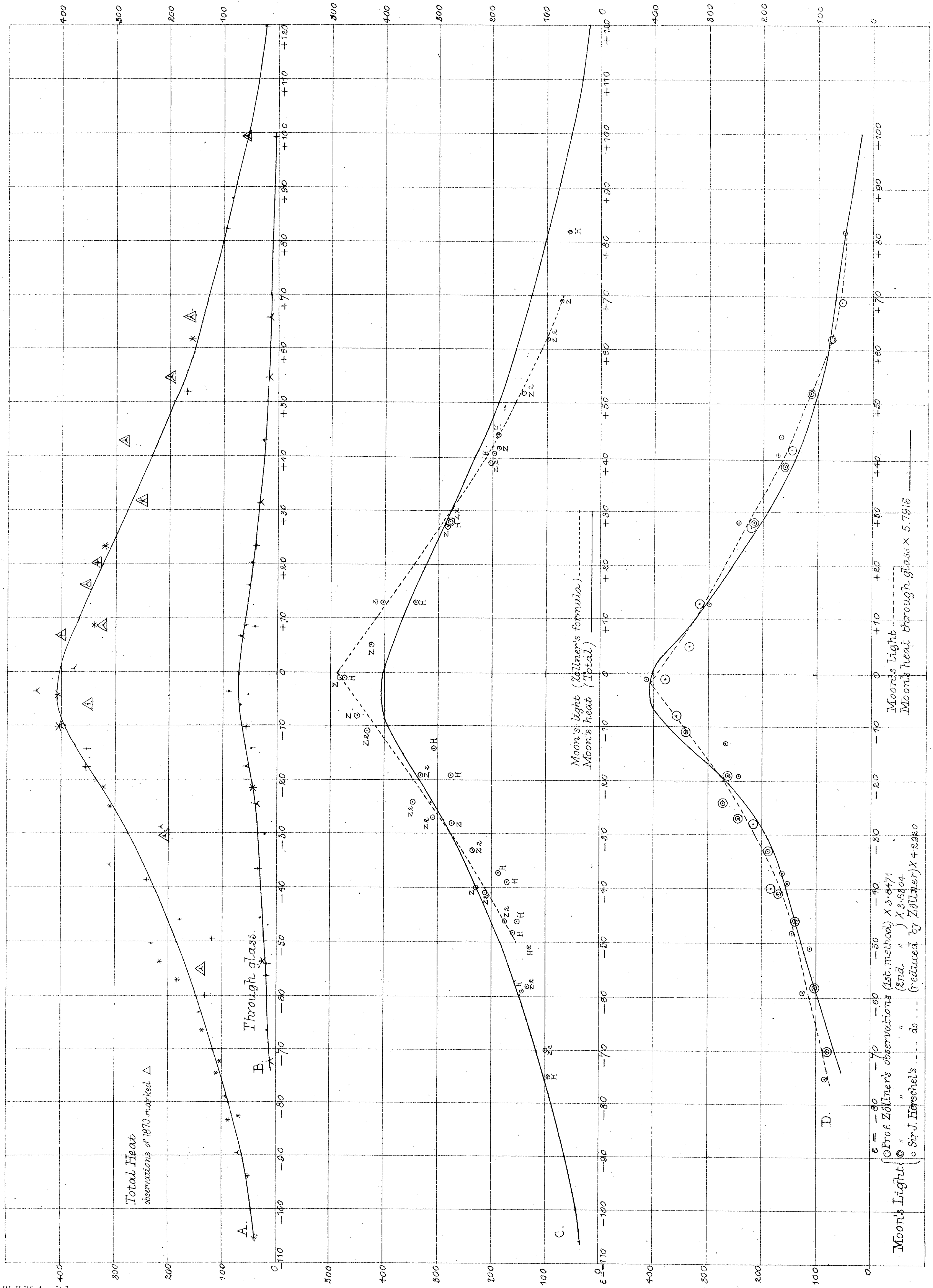
Let R, R′ be respectively the light and heat regularly reflected.

D, D′ be respectively the light and heat regularly diffused, unchanged in refrangibility.

E, E′ be respectively the light and heat regularly absorbed and afterwards emitted.

F, F′ be respectively the light and heat regularly radiated after more than one reflection.

As long as $\frac{D+E+F}{D'+E'+F'}=\frac{R}{R'}$ the two curves will be identical, even though R and R′ be a different function of the elongation from $D+E+F$ and $D'+E'+F'$. But if this relation does not hold, which will be the case if $\frac{R}{R'}=\frac{D}{D'}$ for $F'<F$ and $E'=0$, then the phase-curve for R, R′ will govern the resulting curve for total light more than it will do that for total heat.



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